Piston-Driven Shock Wave Test Problem for Validating Magnetohydrodynamic Models in Astrophysics

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Introduction

- Matter under extreme conditions in heavy-ion collisions and astrophysics.
- Developing theoretical and computational methods.
- Presenting the Piston Shock-Driven test problem to validate the computational code.

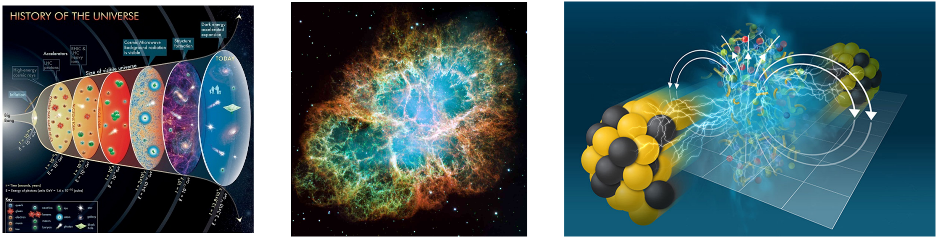

Figure 1: (a): History of the Universe [1, 2], (b): The discovery of a new type of supernova explains a stellar explosion from A.D. 1054 [3], (c): Heavy Ion Collisions [4].

Conservation Laws [4][5]

$$\partial_\mu T^{\mu\nu} = 0 \tag{1}$$

$$\partial_\mu N^\mu = 0 \tag{2}$$

where:

$$T^{\mu\nu} = (T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{EM}}) = (\rho h + p)u^\mu u^\nu + p g^{\mu\nu} - b^\mu b^\nu$$

h is the specific enthalpy, p is the pressure, and $g^{\mu\nu}$ is the metric tensor.

Rankine-Hugoniot Jump Conditions

$$[\![\rho\gamma v]\!] = 0 \quad \text{(Relativistic Mass Conservation)} \tag{3}$$

$$[\![(\rho h + b^2)\gamma^2 v^2 + p + \frac{1}{2}b^2 - (b^x)^2]\!] = 0 \quad \text{(Relativistic Momentum Conservation)} \tag{4}$$

$$[\![(\rho h + b^2)\gamma^2 v - b^0 b^x]\!] = 0 \quad \text{(Relativistic Energy Conservation)} \tag{5}$$

Computational Framework: Relativistic MHD

Equations (1) and (2) can be cast into the following form which can be solved numerically using a computer:

$$\partial_t \mathbf{U} + \partial_x \mathbf{F}(\mathbf{x}) = 0 \tag{6}$$

Here:

$$\mathbf{U} = \begin{pmatrix} D \\ S^x \\ \tau \\ B^y \\ B^z \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} Dv^x \\ S^x v^x + p^* - (b^x)^2 \\ S^x - Dv^x + p^* v^x - b^0 b^x \\ B^y v^x - B^x v^y \\ B^z v^x - B^x v^z \end{pmatrix}$$

U is a set of conservative variables and **F** is a flux vector. $p^* = p + \frac{b^2}{2}$ is the total pressure including magnetic pressure, and b^μ is the magnetic field 4-vector in the fluid frame.

The conservative variables are defined in terms of primitive variables:

$$D = \rho\gamma, \quad S^x = \rho h \gamma^2 v^x + b^2 v^x - b^0 b^x, \quad \tau = \rho h \gamma^2 - p + \frac{b^2}{2} - (b^0)^2 \tag{7}$$

PLUTO Code

- Open-source CFD tool for astrophysical flows.
- Supports 1D, 2D, and 3D simulations.
- Pros: Versatile, efficient.
- Cons: No GR; rarely used in HIC studies.

Figure 2: The PLUTO Code — The PLUTO Code for Astrophysical GasDynamics [6].

Problem Set-up

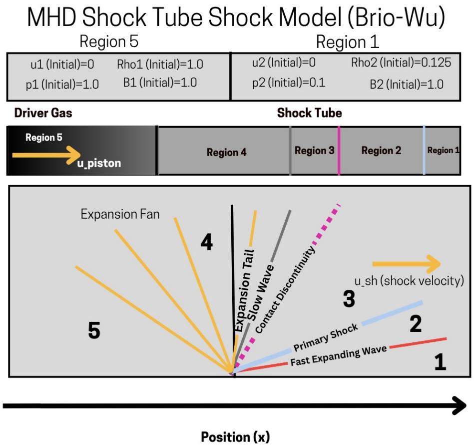

Figure 3: A typical x-t RMHD diagram [5].

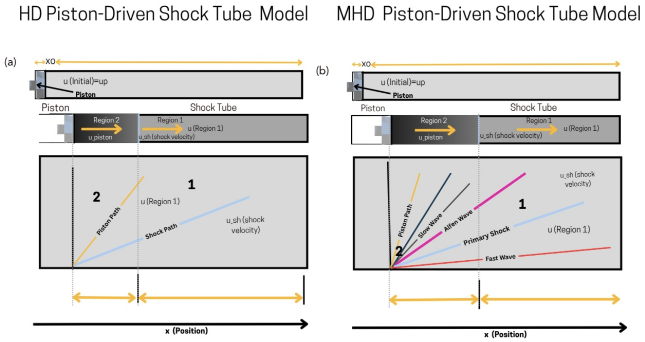

Figure 4: General HD (a) and MHD (b) x-t diagrams [5, 7].

Results: Brio-Wu Shock Tube Model Profiles

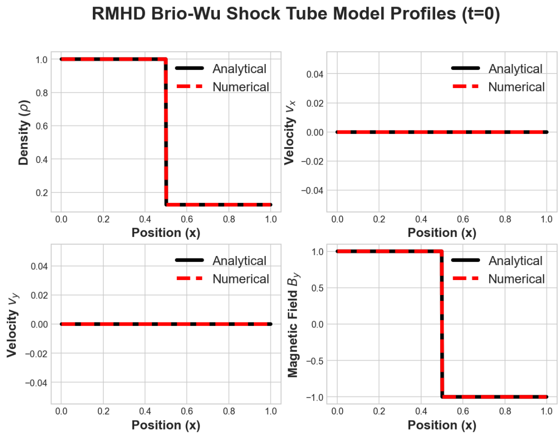

Figure 5: RMHD Brio-Wu Shock Tube Model Initial Conditions Profiles.

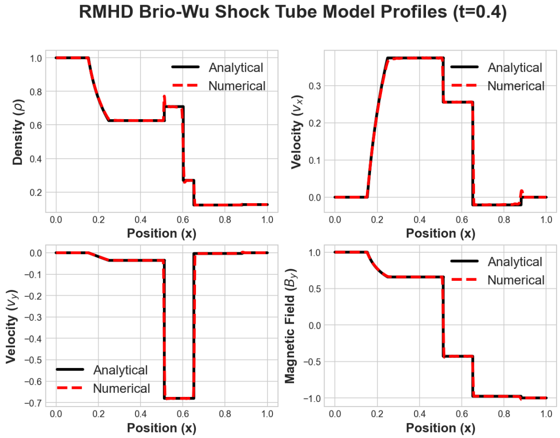

Figure 6: RMHD Brio-Wu Shock Tube Model Profiles at t = 0.4.

Results: HD Piston Driven-Shock Tube Model

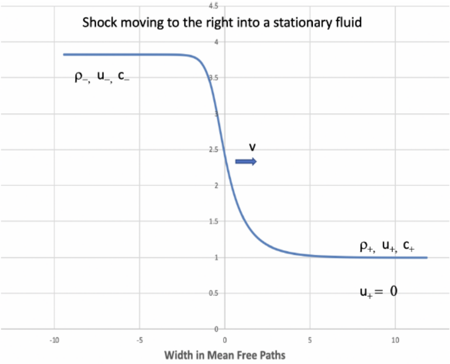

Figure 7: Density profile for piston driven shock moves into a stationary fluid with velocity **v** [8].

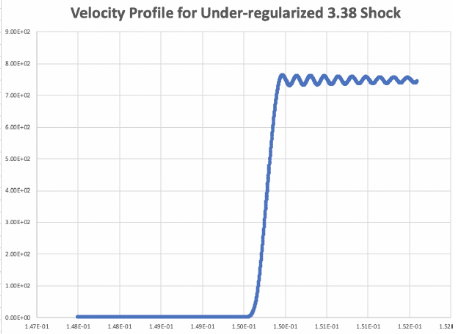

Figure 8: Non-physical oscillations (wiggles) appear when the magnitude of the artificial viscosity is too small [8].

Results: HD Piston Driven-Shock Tube Model

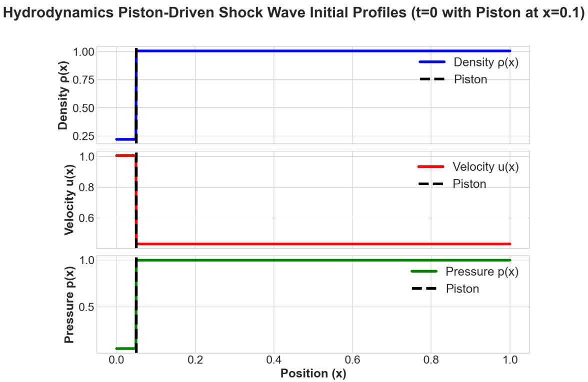

Figure 9: Hydrodynamics Piston-Driven Shock initial conditions profiles (t= 0).

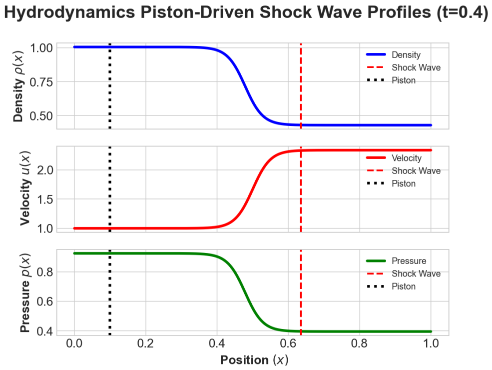

Figure 10: Hydrodynamics Piston-Driven Shock Wave Profiles (t=0.4).

Results: MHD Piston Driven-Shock Tube Model

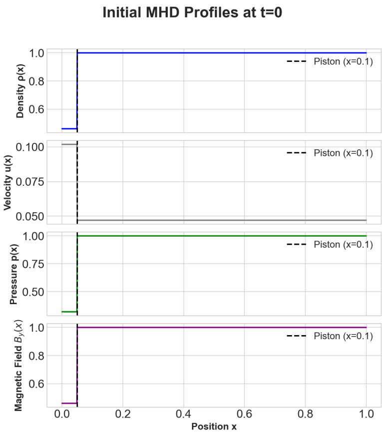

Figure 11: 1D MHD Piston-Driven Shock Model initial conditions profiles (t= 0.0).

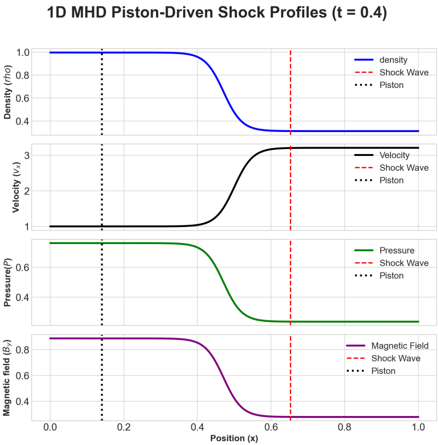

Figure 12: 1D MHD Piston-Driven Shock profiles (t= 0.4).

Discussion (Continued)

Relativistic Magnetohydrodynamics (RMHD)

- Describes plasma–magnetic field interactions in extreme astrophysical settings.
- Grounded in conservation of stress-energy and particle number.
- Accounts for both thermal and magnetic pressures; applies Rankine–Hugoniot jump conditions across shocks.

- Piston model simulates the bounce phase in core-collapse supernovae, capturing high densities, velocities, and magnetic fields.
- Plasma-β highlights the balance between thermal and magnetic pressures.

Discussion

Computational Validation – PLUTO Code

- PLUTO: Open-source code for simulating 1D/2D/3D astrophysical flows.
- Validated using the Brio–Wu test, ensuring reliable shock-capturing performance.

Physics of Wave Generation and Propagation:

- In MHD, these interactions produce complex wave patterns.

Conclusion

- PLUTO Code was used to generate state profiles to study the Brio-Wu shock tube test problem for RMHD validation.
- Comparison of Analytical and Numerical Results was undertaken.
- Assessment of Shock-Capturing Capability of the Piston-driven shock tube model.

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