A first approach to modelling cosmic ray diffusion coefficients in the presence of synthetic compressive magnetic turbulence in the heliosheath

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GCR modulation in the heliosheath

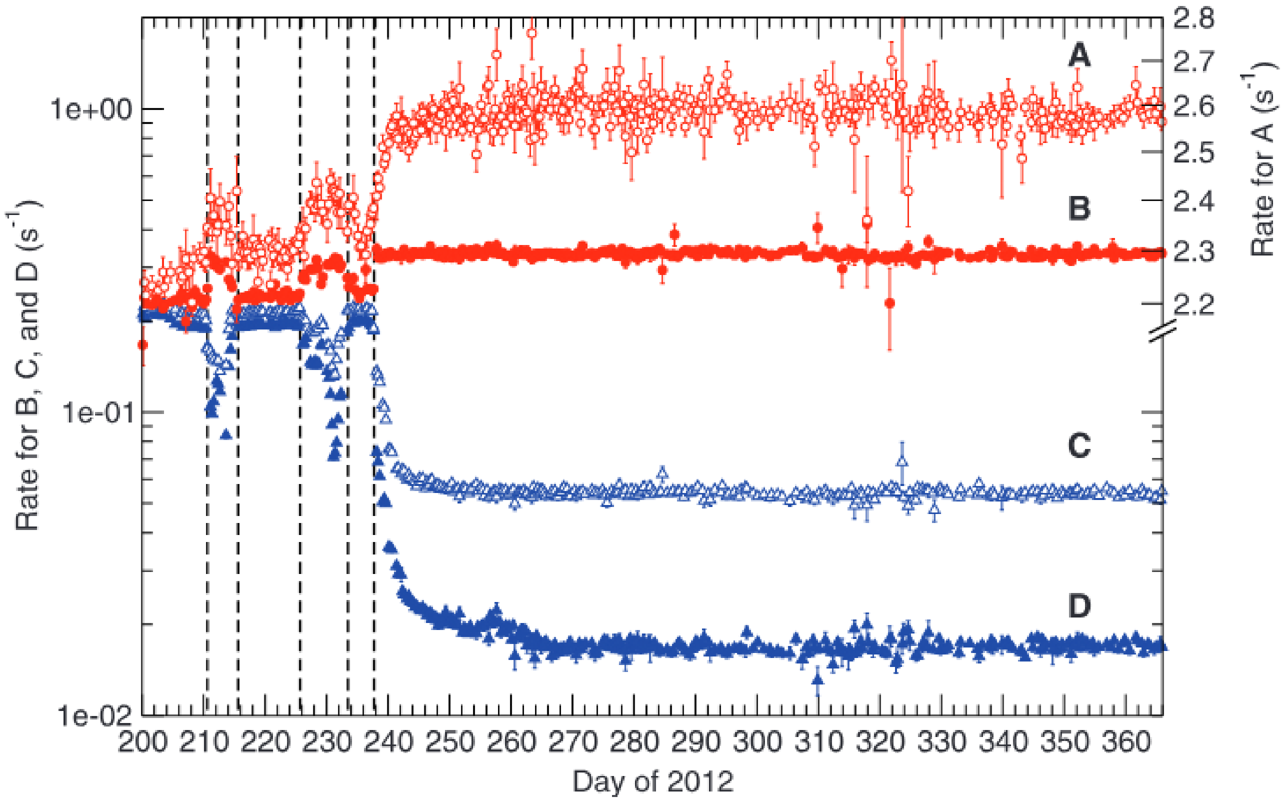
- During August 2012, the Voyager 1 spacecraft observed an abrupt, significant decrease in the intensities of cosmic rays (CRs) from within the heliosphere as it traversed the heliosheath (HS).

- This decrease in comparatively low-energy anomalous cosmic rays was accompanied by a corresponding increase in high-energy nuclei originating from beyond the heliosphere (see figure on next slide, taken from Stone et al., 2013).

- The enhanced galactic cosmic ray (GCR) intensities were detected while magnetic field measurements indicated that Voyager 1 was still within the heliosphere.

- Such observations imply that considerable CR modulation occurs within the HS.

GCR modulation in the heliosheath



- Lines A and B (red) show observed intensities for GCR protons (with energies larger than 70 MeV) and electrons (with energies between 6 and 100 MeV), respectively.

- Lines C and D (blue) show observations of lower-energy particle intensities; C corresponds mainly to anomalous CR protons with energies between 7 and 60 MeV before 25 August, and to GCR protons in this energy range after, while D mainly depicts the intensities of protons between 0.5 and ~30 MeV accelerated at the termination shock and in the HS.

Image taken from Stone et al., 2013.

GCR modulation in the heliosheath

- Modulation studies have found that the GCR intensities observed in the HS may be accounted for by altering the ratio between the parallel and perpendicular diffusion coefficients ($\kappa_{\parallel}/\kappa_{\perp}$).
- Given a background magnetic field directed along the *z*-axis, these diffusion coefficients are given by

$$\kappa_{\parallel} = \lim_{t\to\infty} \frac{\left\langle (\Delta z)^2 \right\rangle}{2t};$$

$$\kappa_{\perp,1} = \lim_{t\to\infty} \frac{\left\langle (\Delta x)^2 \right\rangle}{2t}; \ \kappa_{\perp,2} = \lim_{t\to\infty} \frac{\left\langle (\Delta y)^2 \right\rangle}{2t}.$$

GCR modulation in the heliosheath

- In particular, Luo et al. (2015) report that Voyager observations may be fitted by scaling the ratio between the parallel and perpendicular mean free paths (MFPs; $\lambda_\parallel$ and $\lambda_\perp$, respectively), which are directly proportional to the corresponding diffusion coefficients, in the HS versus in the local interstellar medium (LIM) by a factor of 10^{10}.

- However, it is unclear how the above diffusion coefficients should be modelled, as turbulence in the heliospheric magnetic field (HMF) in the HS differs significantly from conditions observed in the inner heliosphere, and thus from what has guided the majority of HMF turbulence studies to date.

- One key characteristic of HS turbulence is the presence of a compressive component (see, e.g., Burlaga, 1994; Burlaga et al., 2006; Burlaga et al., 2015; Zhao et al., 2019; Fraternale et al., 2022), which corresponds to fluctuations parallel to the background magnetic field.

- The above forms the basis for the work presented here: a preliminary investigation of GCR transport effects which may arise from magnetic turbulence conditions found in the HS, using turbulence parameters based on available HS analyses as inputs for the particle pusher code detailed by Els & Engelbrecht (2024).

Basic properties of the heliosheath

- The heliosheath is the region consisting of a subsonic, shocked plasma situated between the termination shock, which is found an average heliocentric distance of the order of 90 au (e.g., Stone et al., 2005; Stone et al., 2008) and the heliopause, found at approximately 120 au from the Sun (e.g., Stone et al., 2013; Burlaga et al., 2019).

- Estimated to cover a distance of 30-40 au in the “nose” direction of the heliosphere (that is, opposite to the direction of the LIM flow), the HS may extend to considerably larger distances downstream of the interstellar wind flow (e.g., Richardson & Stone, 2009).

- Based on these estimates, we assume a length of 30 au for the HS, although this can, of course, be varied, especially when the HS downstream of the LIM is considered.

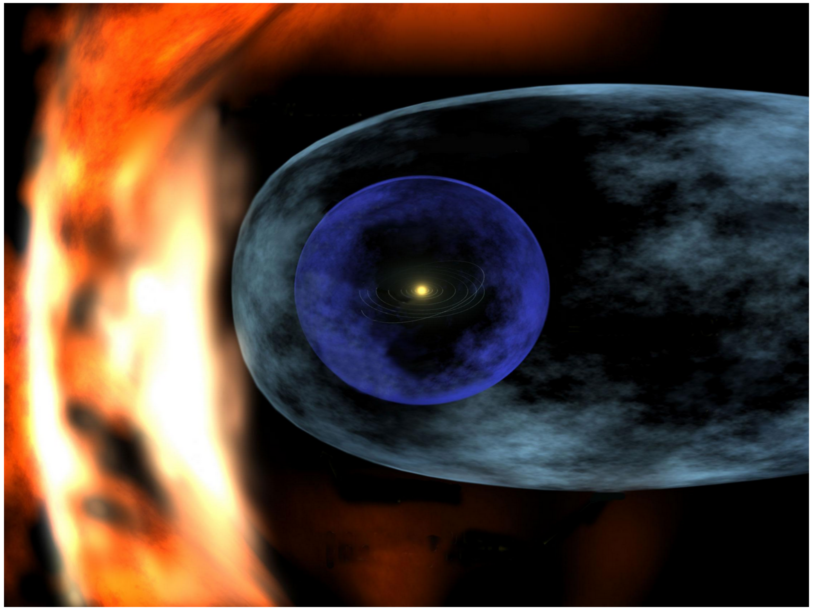


Image credit: NASA/Walt Feimer

Characteristics of HS turbulence

- Some studies have suggested that magnetic turbulence in the HS can be characterised as isotropic, or nearly so (see, however, Fraternale et al., 2019), with an average magnetic field strength of approximately 0.1 nT (e.g., Burlaga et al., 2006).

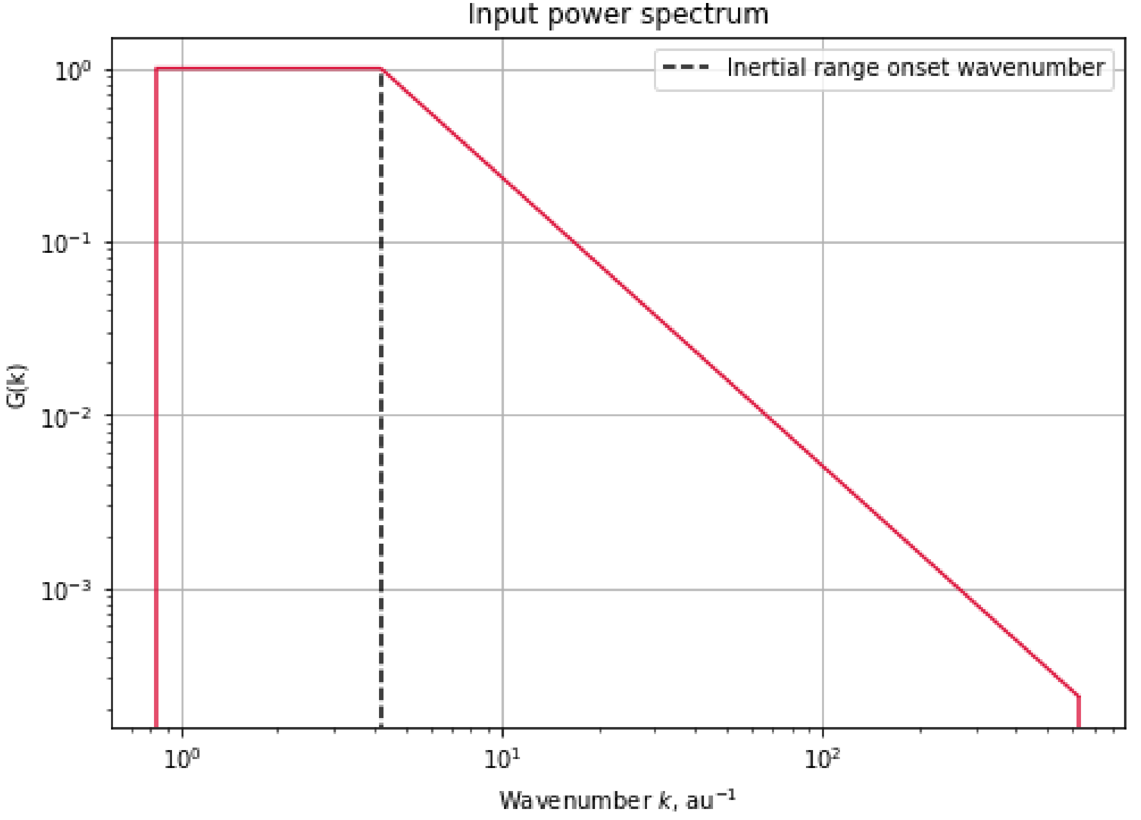
- Additionally, it is worth noting that the standard Taylor hypothesis may not be applicable in the HS, due to the subsonic plasma flow speed (see Zhao et al., 2024), and that its application in the context of the HS may result in incorrect conclusions drawn from Voyager turbulence observations.

- Using an extended approach to analysing Voyager turbulence data, Zhao et al. (2024) conclude that magnetic fluctuations in the HS are consistent with a spectral model which is nearly isotropic, with approximately 25% power allotted to compressive fluctuations.

A simple HS power spectrum

- Based on a detailed analysis of HS turbulence across several fluctuation regimes by Fraternale et al. (2019), we select a maximum fluctuation length of 7.5 au; although these authors report that the largest-scale fluctuations may be of the order of 10 or 20 au, these are only rough estimates, and we account for computational constraints, as well as the chosen HS length of 30 au.

- Moreover, we select a minimum fluctuation length of 0.01 au. While the analysis above shows that this length is associated with a spectral break, we consider only a flat energy-containing and a Kolmogorov inertial range, with an onset at 1.5 au (also from the above analysis), in our preliminary modelling.



Modelling magnetic fluctuations

- In order to model compressive fluctuations, a special case of the method proposed by Giacalone & Jokipii (1999), as modified by Tautz & Dosch (2011), is considered.

- The above method is hereafter referred to as the regenerative method. This method involves the summation of a number of plane waves with random directions, polarisation angles, and phases. Angles defining each wavemode are initially randomly selected, but thereafter kept fixed for the modelling of a given turbulence realisation.

- The result is an analytic description of the magnetic field at any spatial coordinate (x, y, z), with zero divergence being guaranteed, and without the need for a grid.

- At a given point in space, the fluctuating component of ***B*** is given by

$$\vec{b} = \varepsilon \sum_{n=1}^{N_m} \hat{\xi}_n A(k_n) \cos\left[k_n z' + \beta_n\right],$$

  where ε is a correction factor due to the choice of the cosine function to model fluctuations, and β is the wave phase. The number of wavemodes is given by N_m. Waves are described in primed coordinates, with the polarisation vector of a given wavemode given as follows:

Modelling magnetic fluctuations

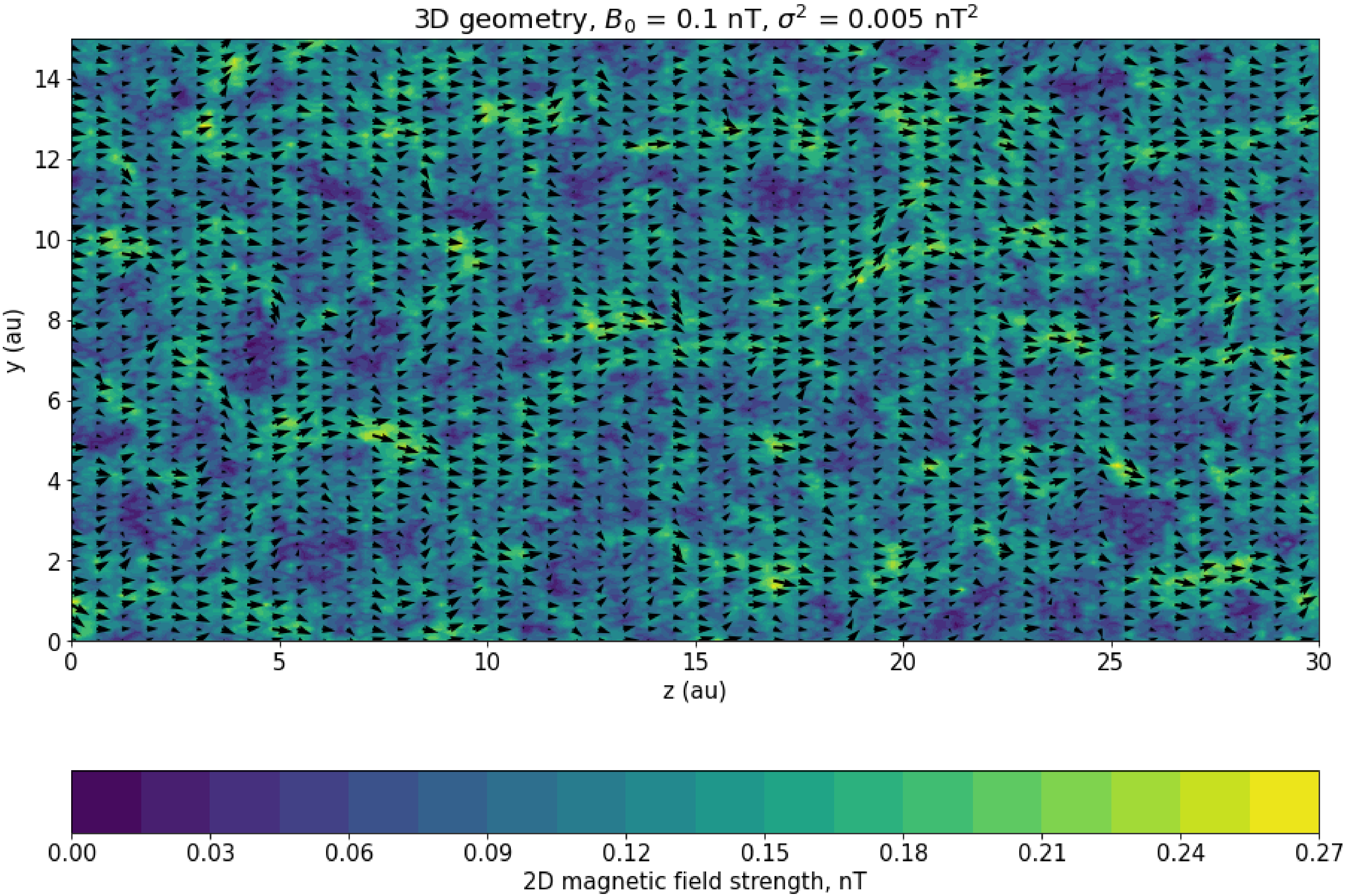
$$\hat{\xi} = \begin{bmatrix} -\sin\phi\cos\alpha + \eta\cos\phi\sin\alpha \\ \cos\phi\sin\alpha + \eta\sin\phi\sin\alpha \\ -\sqrt{1-\eta^2}\sin\alpha \end{bmatrix}.$$

- The angles *φ*, *α*, and *β* may, for isotropic turbulence, be chosen at random, between 0 and 360°.
- The quantity *η* may be chosen in the range [-1, 1].
- Finally, the amplitude function is given by

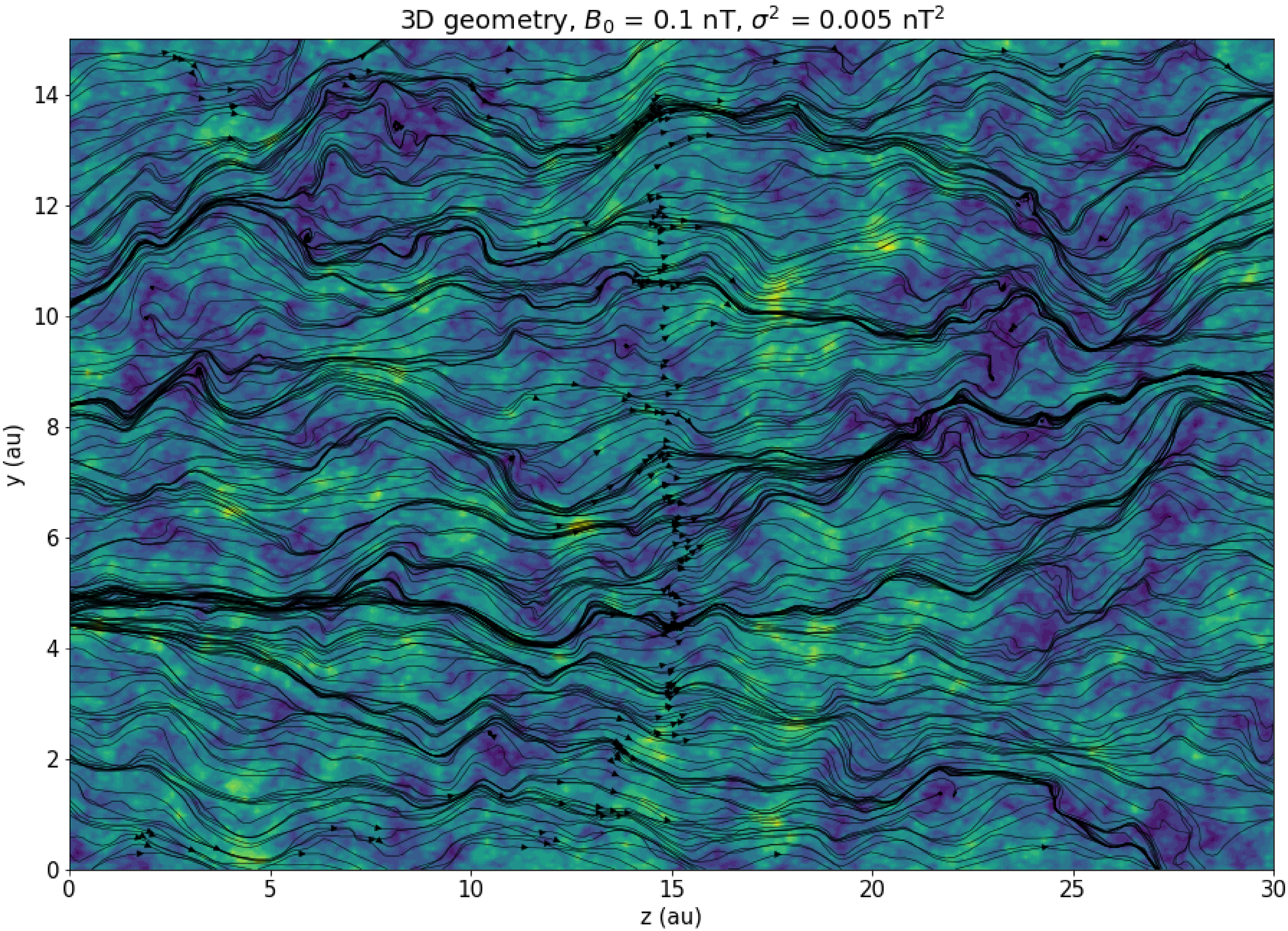
$$A^2(k_n) = G(k_n)\Delta k_n \left(\sum_{\nu=1}^{N_m} G(k_\nu)\Delta k_\nu \right)^{-1},$$

where the wavenumbers are assumed to be spaced logarithmically, with spacing *Δk/k* a constant. The function *G* is the input power spectrum, shown earlier.

Modelling magnetic fluctuations



Modelling magnetic fluctuations



Modelling magnetic fluctuations

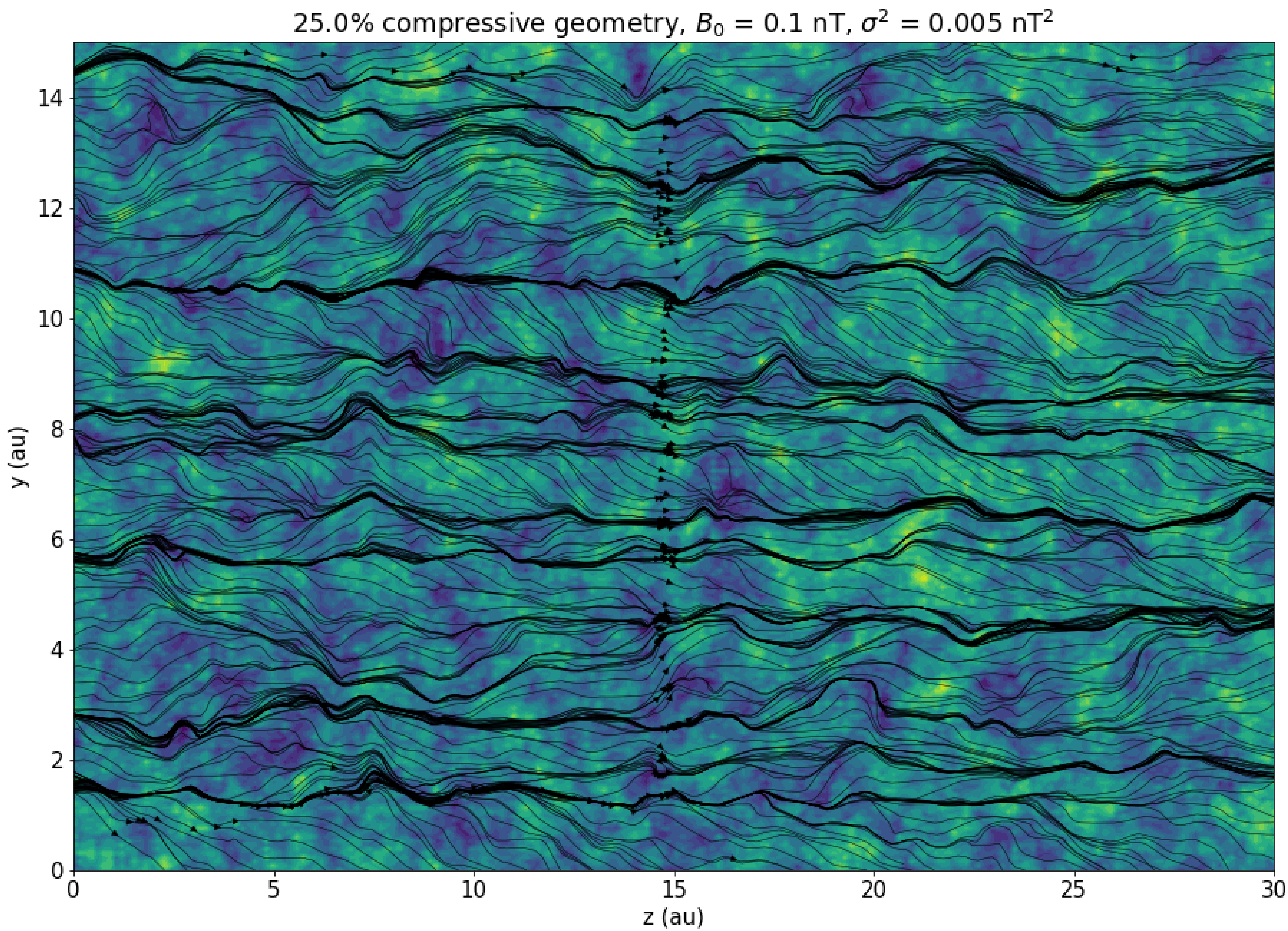
- Specific turbulence geometries may be obtained by fixing the values of certain angles. For example, slab turbulence (e.g., Jokipii, 1966) may be modelled by letting η=1; this corresponds to fluctuations propagating exclusively along the background magnetic field, assumed to be directed along the *z*-axis.

- For this study, we consider in particular a special case of the 2D geometry, such that η=0. Had the 2D turbulence (e.g., Matthaeus et al., 1990; Matthaeus et al., 1995; Bieber et al., 1996) been transverse, the value of α would have been chosen as zero, so that fluctuations are perpendicular to the background field; however, for fluctuations in *B* directed *along* the *z*-axis, one may, in principle, let α=90°. Such fluctuations, which are considered in conjunction with slab and 2D transverse turbulence, compress the background field, and would vary with *x* and *y* only.

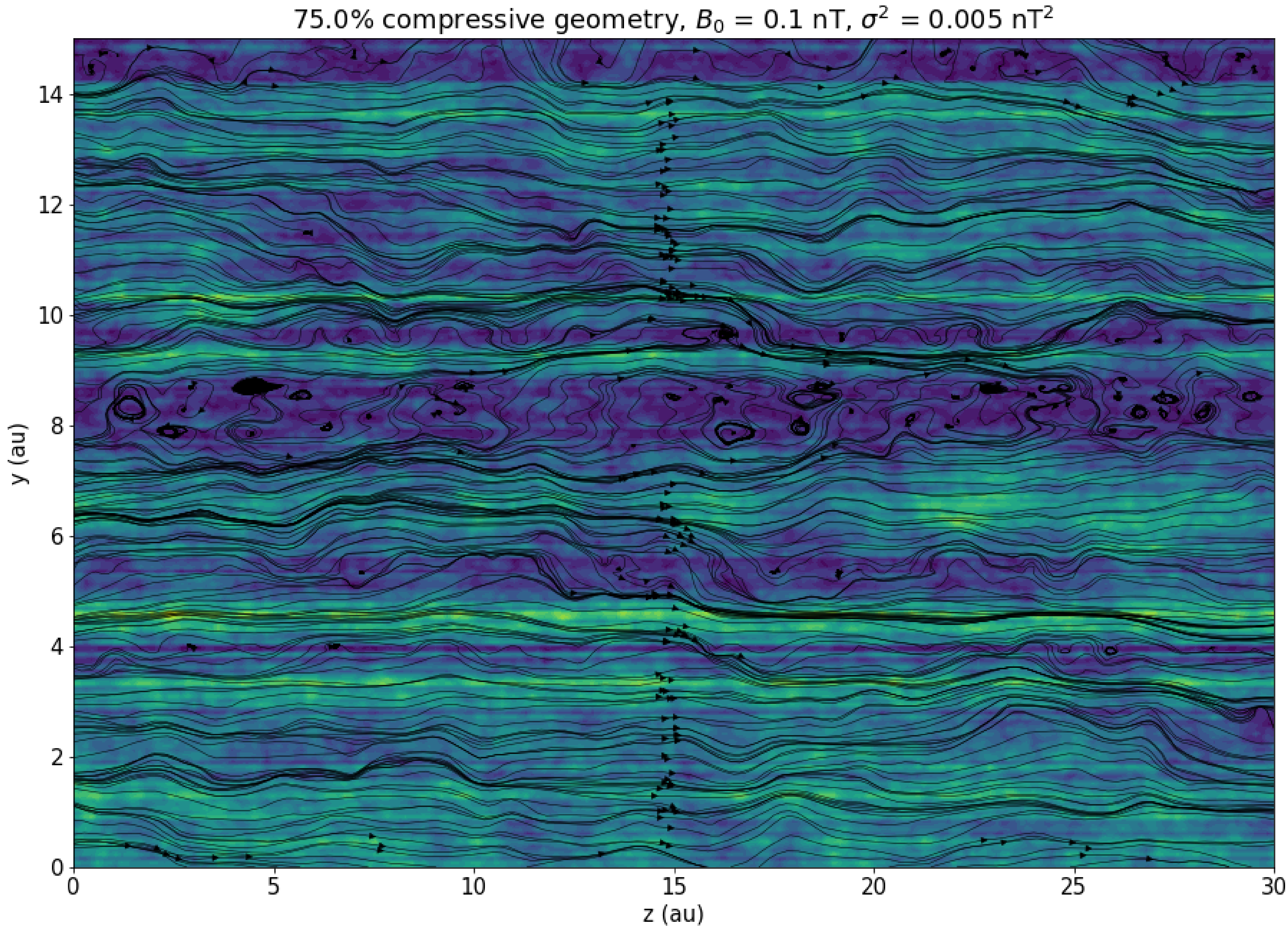
- In the interest of computational time and resolution, we model transverse (slab + 2D) turbulence by employing the grid-based method as implemented by, e.g., Minnie et al. (2007), and Els & Engelbrecht (2024). The input magnetic variance is then divided between the regenerative compressive component and (evenly) between the grid-based transverse components, with different values chosen for the ratio $(dB_z/dB_T)^2$.

$$\vec{b}_{transverse} = \left(\vec{b}(z)_{slab} + \vec{b}(x,y)_{2D}\right);\ \vec{b}_{compressive} = \vec{b}(x,y)\hat{z}$$

Modelling magnetic fluctuations



Modelling magnetic fluctuations



Faux-isotropy: a test case

- As a rough first measure of the viability of this three-component approach, we consider briefly GCR diffusion in the faux-isotropic case, in which variances are divided evenly between b_x, b_y, and b_z, and the assumed power spectrum is the same for the three components. Here, no background field is assumed.

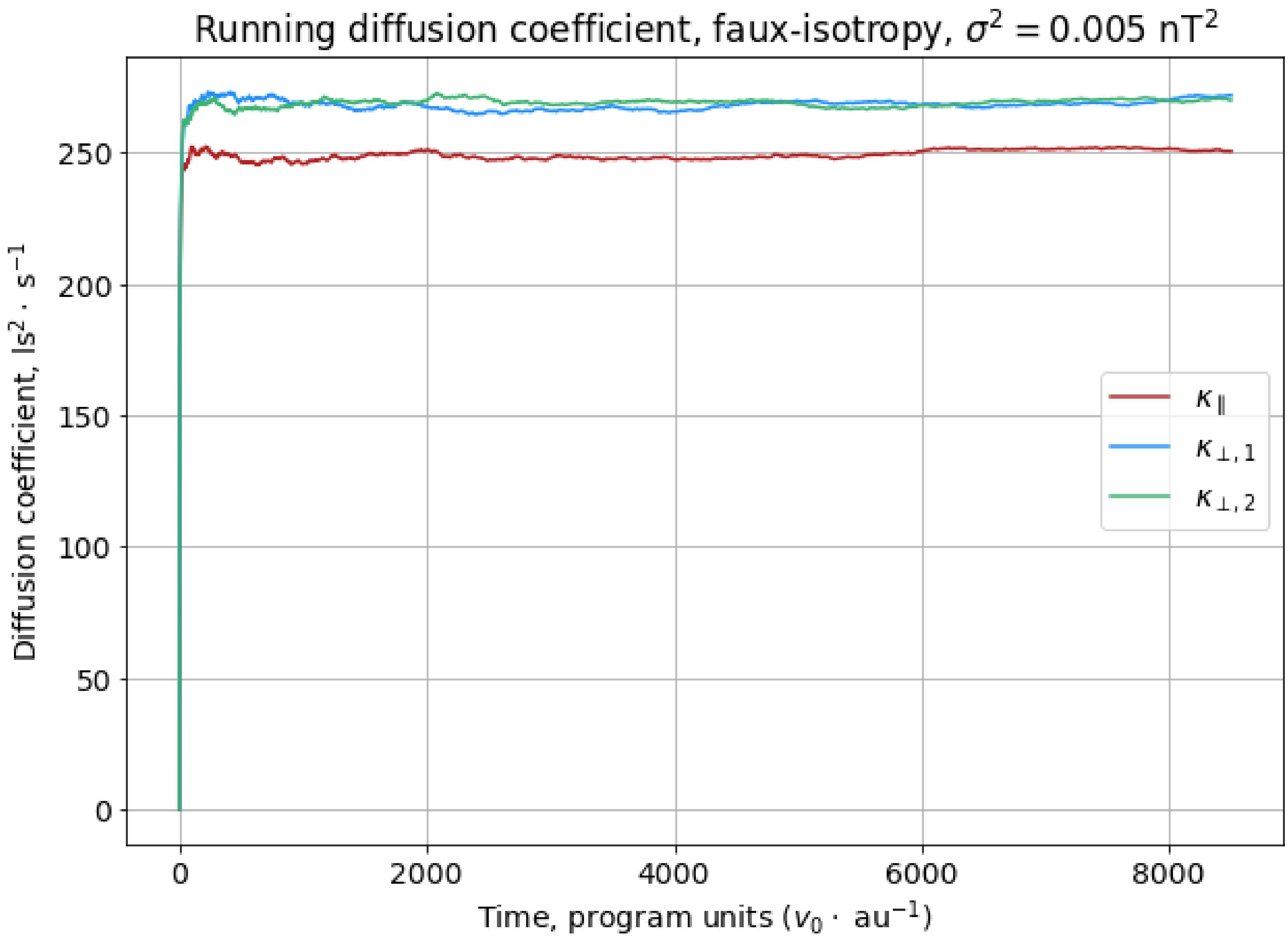
- Of course, in the case of true isotropy, all of the components of the fluctuating field would vary in all three spatial dimensions; for the present approach, the compressive component varies only along two spatial dimensions (x and y), whereas the (composite) transverse turbulence varies with x, y, and z, owing to its 2D as well as slab turbulence contributions.

- Moreover, the parallel diffusion coefficient and two perpendicular diffusion coefficients would converge to the same value in the case of isotropic turbulence without a guiding field.

- It is encouraging, then, that for an ensemble-averaged test run of the faux-isotropic case, the parallel and perpendicular diffusion coefficients are found to be comparable, differing by less than 10% on average.

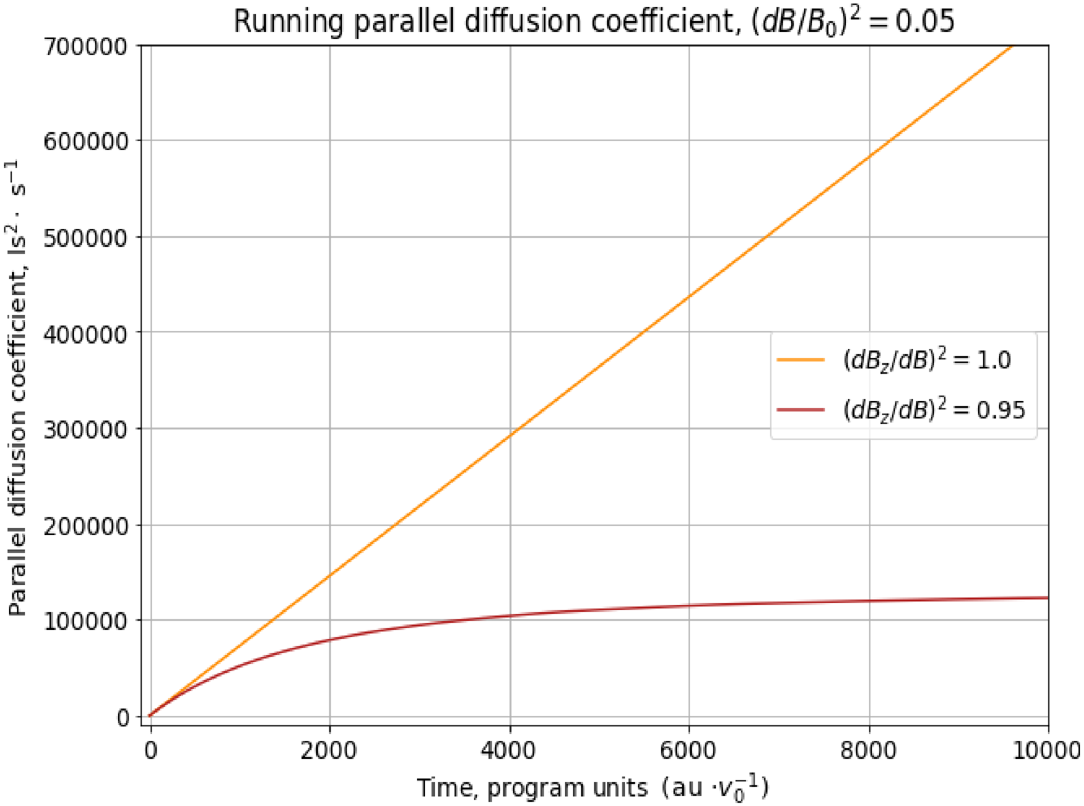
- The exact circumstances under which such faux-isotropy is a reasonable approximation will be explored further in future work. For the remainder of this presentation, turbulence configurations with varying compressive contributions are considered.

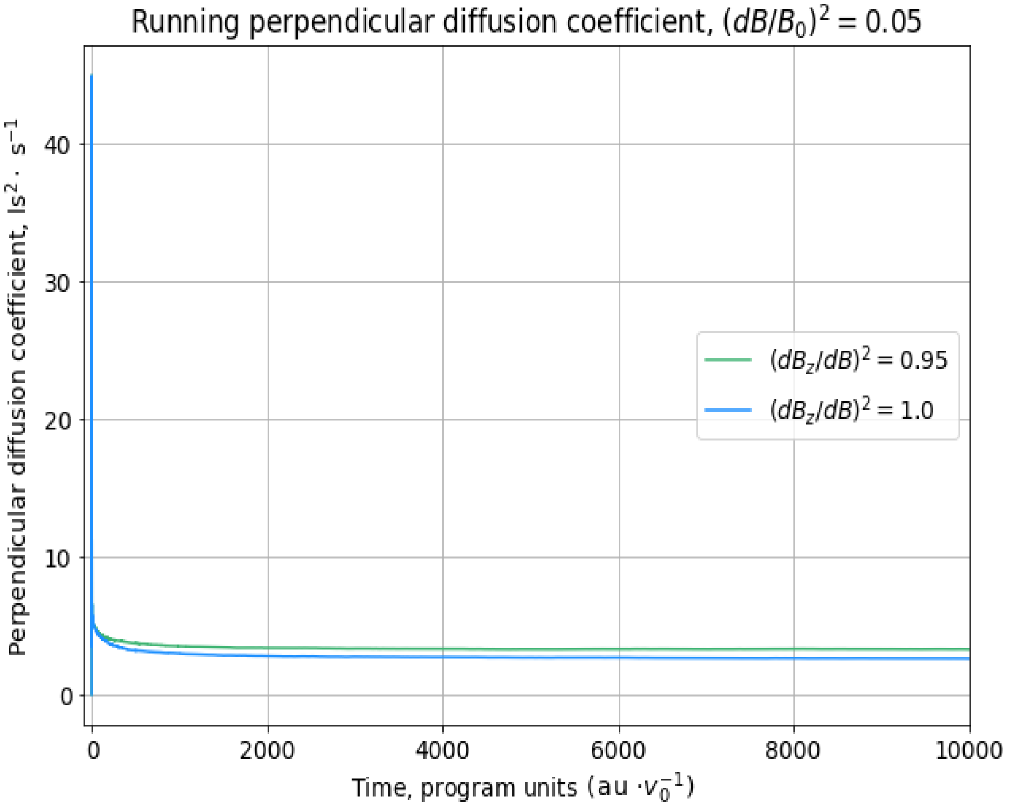
Faux-isotropy: a test case



Strongly compressive turbulence

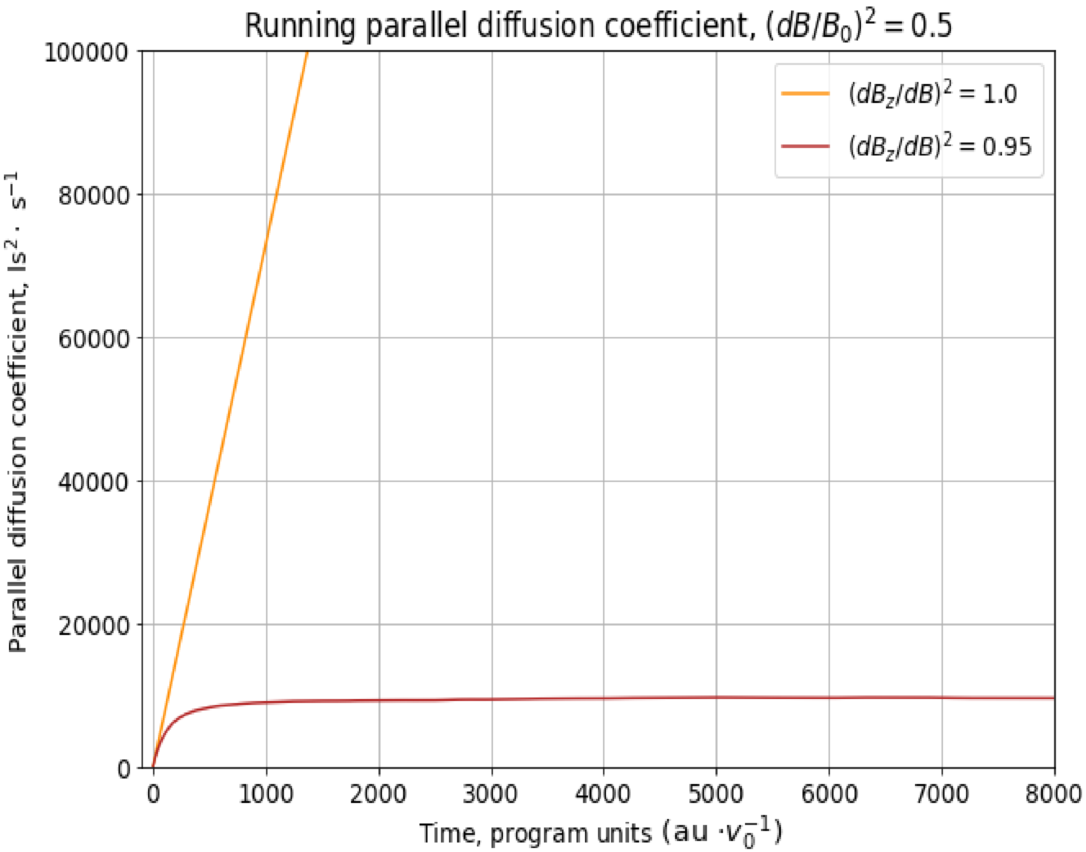
- In the results presented here, we consider particle transport in the presence of magnetostatic turbulence with a large compressive component.
- The running diffusion coefficients of 1 GeV protons computed for a low turbulence level are shown below. The units shown on the horizontal axis are program units; in the context of this investigation, this unit is the time elapsed while a 1 GeV proton traverses a distance of 1 au.

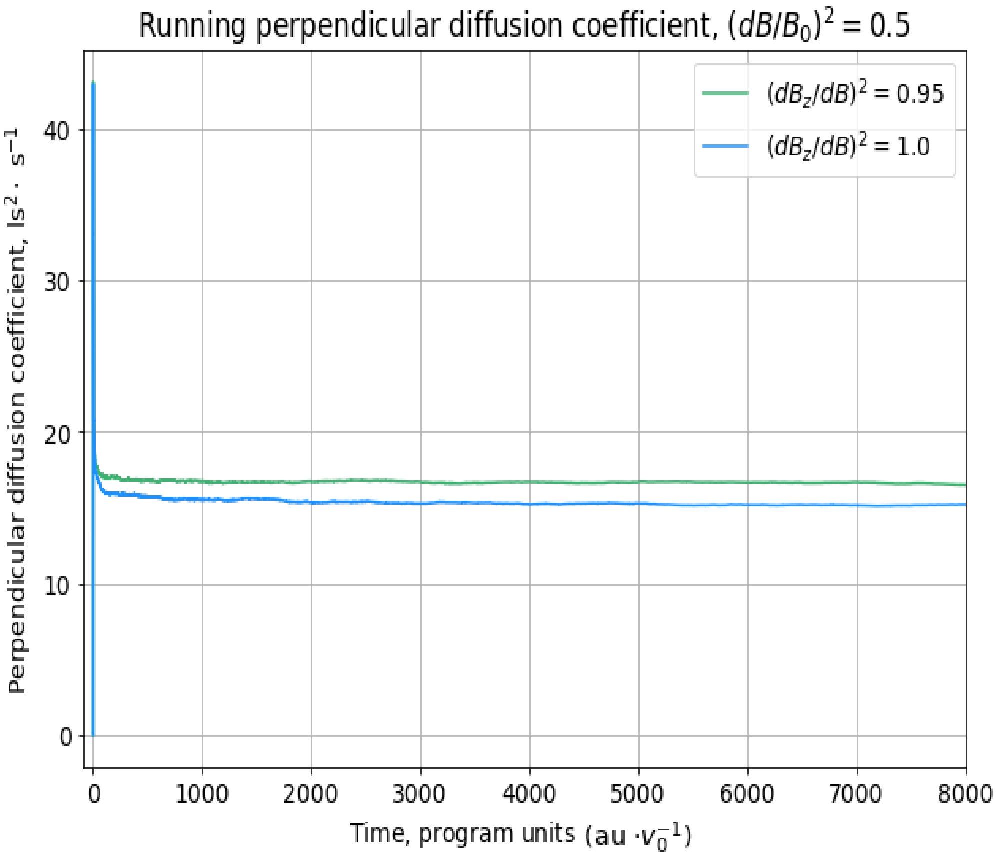




Strongly compressive turbulence

- At higher turbulence levels, free streaming is still observed in the case of purely compressive turbulence, and an overall increase in the perpendicular diffusion coefficient is seen. The addition of a transverse component here results in a parallel diffusion coefficient which is reduced considerably from the low-turbulence case.

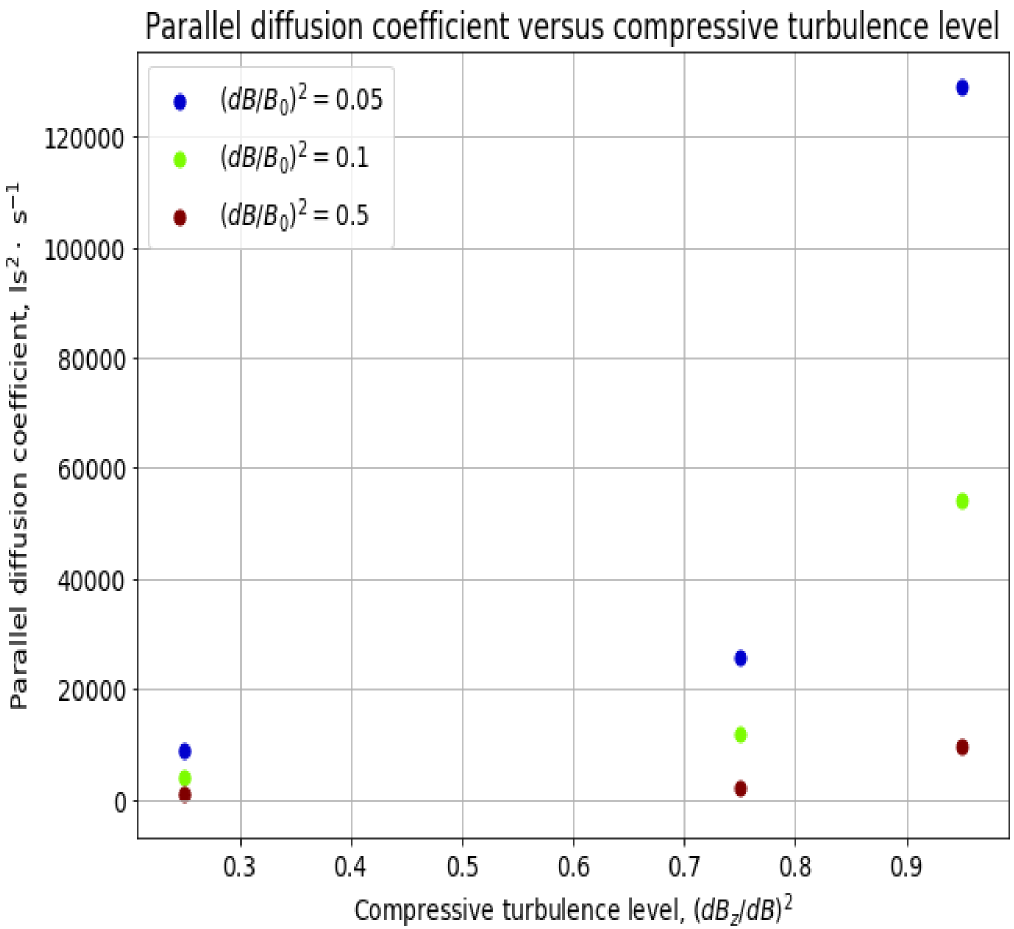


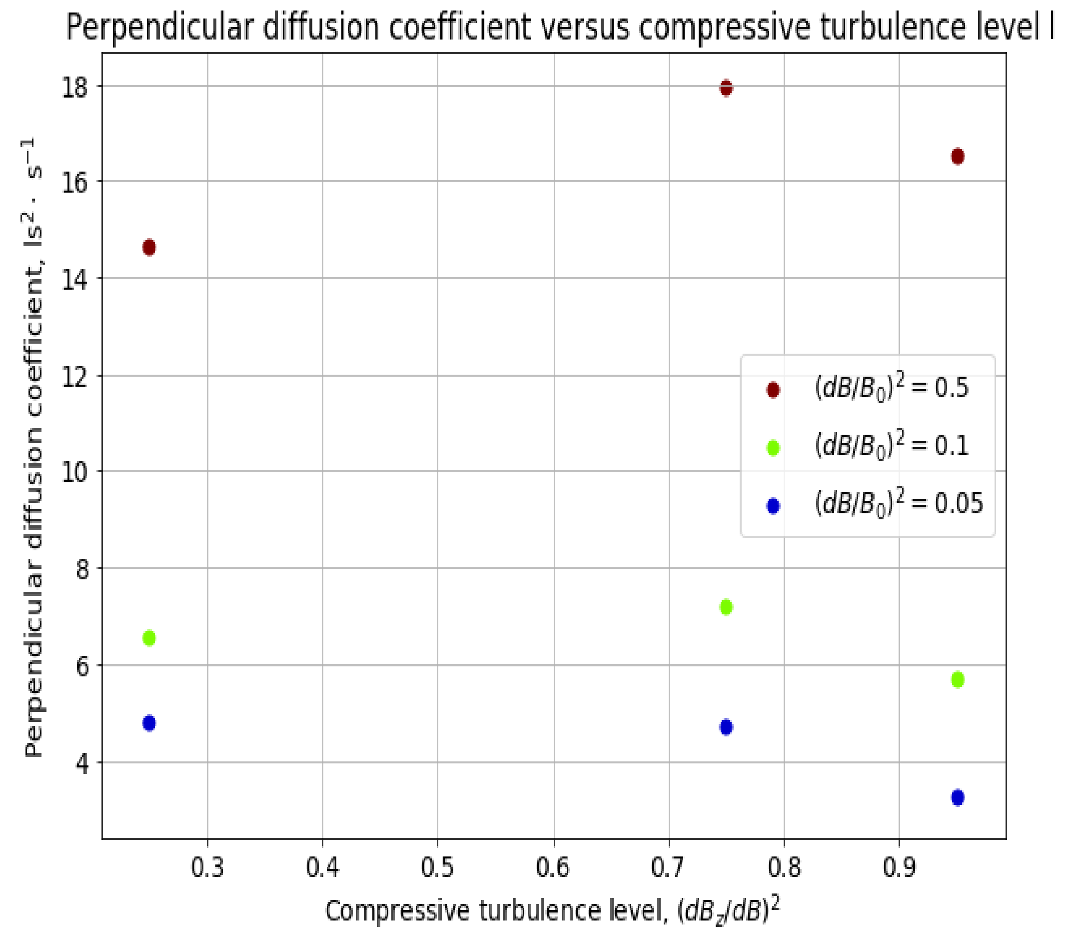


Diffusion due to compressive turbulence

- In the following figures, we consider three different values for the turbulence level $(dB_T/B_0)^2$: 0.5, 0.1, and 0.05.

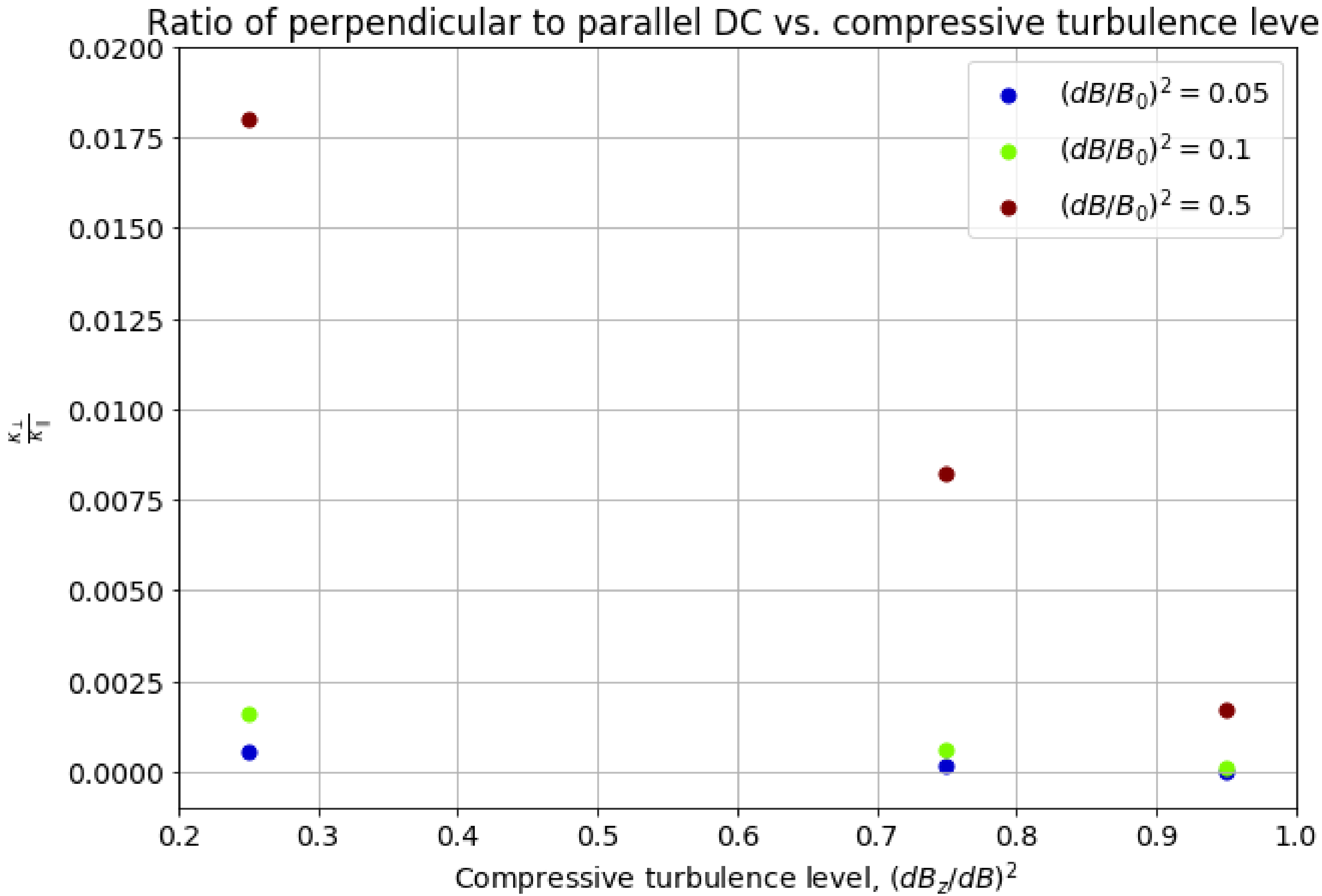
- The contribution of the compressive component is varied. We consider turbulence with a large compressive contribution, such that $(dB_z/dB_T)^2$=0.95, as well as the cases of $(dB_z/dB_T)^2$=0.75 and $(dB_z/dB_T)^2$=0.25, with the last-mentioned being motivated by the percentage estimate reported by Zhao et al. (2024).





Diffusion due to compressive turbulence

- One may also consider the ratio between perpendicular and parallel diffusion coefficients computed for different levels of compressive turbulence.



Conclusions and future work

- While the ratio between the perpendicular and parallel diffusion coefficients is seen to be small, especially at lower turbulence levels, it does not appear to differ considerably from what is expected at 1 au. It would therefore seem as though another mechanism may be responsible for the high levels of modulation observed in the heliosheath.

- Data presented here are sparse; while clear trends with regard to the behaviour of the parallel diffusion coefficient can be seen, the effect of compressive turbulence on the perpendicular diffusion coefficient is less clear. More simulations for other levels of turbulence are needed.

- The influence of a z-dependent compressive component will also need to be investigated, as will the differences between real isotropy and the faux-isotropic case considered here. In particular, the limits of the applicability of faux-isotropy will need to be ascertained.

Acknowledgements

- I would like to thank the South African National Space Agency for funding my studies.

- For any further questions or a full list of references, do not hesitate to email me at jst99960@gmail.com.

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