Testing f (Q) gravity as a solution for H_0 Tension

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General Relativity (GR)

- GR: The standard theory of gravity—geometric property of four-dimensional spacetime (Einstein 1915).
- $S_{EH} = \frac{1}{2\kappa} \int [R + 2\kappa \mathcal{L}_M] \sqrt{-g}\, d^4x$, where $\kappa = \frac{8\pi G}{c^4}$
- $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$, where $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$

Successes

- Describes gravitational interactions across a wide range of scales.
- Predicts expansion and structure formation.

Friedmann and Raychaudhuri Equations

- perfect fluid $T_{\mu\nu} = (\rho + P) u_\mu u_\nu + p g_{\mu\nu}$
- FLRW metric $ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$
- $8\pi G = c^4 = 1$

$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3}$ and $\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P)$

Shortcomings and ΛCDM

Shortcomings

- Late-time accelerated expansion (SNIa)
- Early structure formation (high-redshift galaxies)

ΛCDM: Standard Cosmological Model

- $S_{EH} = \frac{1}{2} \int \left[(R - 2\Lambda) + \mathcal{L}_M \right] \sqrt{-g}\, d^4x$
- $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$

Friedmann Equations with Λ

- $H^2 = \frac{1}{3}\rho + \frac{\Lambda}{3}$
- $\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P) + \frac{\Lambda}{3}$

ΛCDM Problems and Solutions

ΛCDM Problems

- Fine-tuning: $\rho_{\text{vac}} \gtrsim 10^{21} \rho_\Lambda$
- Coincidence problem: $\rho_m \sim \rho_\Lambda$ today
- Unknown nature of dark sector
- H_0 tension: 67.4 ± 0.5 vs 73.04 ± 1.04 (5σ)
- S_8 tension: 0.831 ± 0.013 vs $0.766^{+0.020}_{-0.014}$ (3.1σ)

Possible Solutions

- Dynamical dark energy ($\rho_{DE} = \alpha H + \beta H^2$)
- Interacting dark sector fluids ($Q = 3b^2 H \rho_m^\delta \rho_{DE}^\gamma \rho_{tot}^\sigma$)
- Modified gravity: $S = \frac{1}{2} \int [f(Q) + 2\mathcal{L}_M] \sqrt{-g}\, d^4x$

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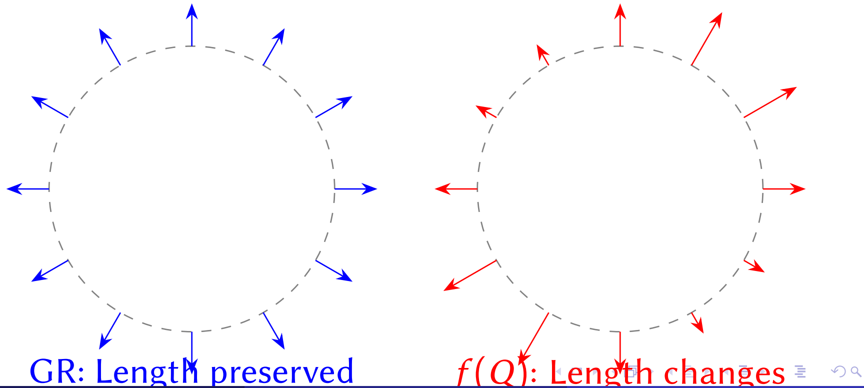
$f(Q)$ Cosmology

- Based on symmetric teleparallel gravity (curvature and torsion vanish)
- Gravity described via nonmetricity tensor $Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu} \Rightarrow Q$ (nonmetricity scalar) constructed to reproduce GR when $f(Q) = Q$
- useful for modeling (dark enrgy, late-time acceleration, and early universe inflation in some models)
- Vectors change their length and angle under parallel transport



2nd Order Field Equations

$$\frac{2}{\sqrt{-g}}\nabla_\alpha\left(\sqrt{-g}f_Q P^\alpha{}_{\mu\nu}\right) - \frac{1}{2}g_{\mu\nu}f - f_Q\left(P_{\mu\alpha\beta}Q_\nu{}^{\alpha\beta} - 2Q_{\alpha\beta\mu}P^{\alpha\beta}{}_\nu\right) = T_{\mu\nu}$$

Definitions

- $Q = -Q_{\alpha\mu\nu}P^{\alpha\mu\nu}$ (nonmetricity scalar)
- $P^\alpha{}_{\mu\nu} = \frac{1}{4}\left(-Q^\alpha{}_{\mu\nu} + 2Q_{(\mu}{}^\alpha{}_{\nu)} - Q^\alpha g_{\mu\nu} - \tilde{Q}^\alpha g_{\mu\nu}\right)$ superpotential
- $Q_\alpha = Q_\alpha{}^\mu{}_\mu$, $\tilde{Q}_\alpha = Q_\mu{}^\mu{}_\alpha$

$f(Q)$ Friedmann Equations

- $6H^2 f_Q - \frac{1}{2}f = \rho$
- $(12H^2 f_{QQ} + f_Q)\dot{H} = -\frac{1}{2}(\rho + P)$
- $Q = 6H^2$

Power-law Model: $f(Q) = \alpha + \beta Q^n$

Features

- Solved $f(Q)$ Friedmann Equations numerically and fit it to Pantheon SNIa and OHD data
- match GR when $\alpha = 0$ and $\beta = n = 1$
- match ΛCDM when $\alpha = -2\Lambda$ and $\beta = n = 1$

- $\dot{H} - H(1+z)H'$
- $\rho_m = 3H_0^2\Omega_m$
- $\rho_\alpha = 3H_0^2\Omega_\alpha$

H(z) $\rho = \rho_m$, $P = 0$

- $\Lambda CDM : \boxed{H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})}}$
- $f(Q) = \alpha + \beta Q^n : H(z) = \left[\frac{\Omega_{m0}(1+z)^3 + \frac{\Omega_\alpha}{2}}{(2-\frac{1}{n})}\right]^{\frac{1}{2n}}$

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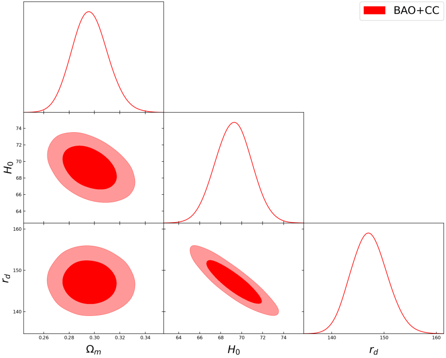

Figure: LCDM

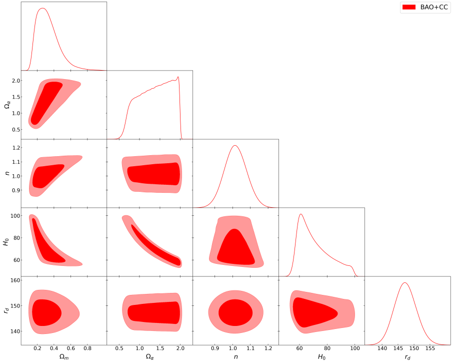

Figure: Model I

MCMC Simulation Free Parameters Constraint

ΛCDM	
	BAO+CC
$\Omega_{m,0}$	$0.296^{+0.014}_{-0.014}$
H_0	$69.230^{+1.713}_{-1.748}$
r_d	$147.113^{+3.537}_{-3.393}$
Model I	
$\Omega_{m,0}$	$0.306^{+0.144}_{-0.107}$
Ω_α	$1.410^{+0.409}_{-0.480}$
n	$1.016^{+0.064}_{-0.062}$
H_0	$68.170^{+15.638}_{-8.194}$
r_d	$147.262^{+3.504}_{-3.366}$

Frame Title

AIC Interpretation (Burnham & Anderson Criterion):

- $\Delta\text{AIC} \leq 2$: Substantial support for the model.
- $4 \leq \Delta\text{AIC} \leq 7$: Considerably less support for the model.
- $\Delta\text{AIC} > 10$: Strong evidence against the model.

BIC Interpretation (Jeffreys' Scale):

- $\Delta\text{BIC} \leq 2$: Weak evidence against the model.
- $2 < \Delta\text{BIC} \leq 6$: Positive evidence against the model.
- $6 < \Delta\text{BIC} \leq 10$: Strong evidence against the model.
- $\Delta\text{BIC} > 10$: Very strong evidence against the model.

Comparison of Statistical Results for Models: ΛCDM vs Model 1

Model	Obs.	Log-Likelihood	χ^2	χ^2_{red}	AIC	BIC	ΔAIC	ΔBIC
ΛCDM	BAO+CC	-13.7241	27.4481	0.7038	35.448	42.493	0.000	0.000
Model 1	BAO+CC	-21.4572	42.9145	1.1293	52.914	61.720	17.466	19.228

Statistical analysis

- **χ^2_{red} Diagnostics:**
  - The reduced chi-squared is above 1, but still within an acceptable range.
  - This suggests a reasonable fit to the data.
  - However, there might be room for improvement in the model or data uncertainties.
- **AIC Interpretation:**
  - ΔAIC indicates strong evidence against the model.
- **BIC Interpretation:**
  - ΔBIC also indicates strong evidence against the model.

Unconstrained Conclusion

- Utilise early universe measurements to constrain the model.
- Perform more extensive tests with large-scale structure formation.
- tests with structure formation and growth rate in future work as an additional probe.

[1–4]

Some of the References

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