Linear polarization measurements on gamma rays from non-oriented nuclear states

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Abstract. Gamma-ray spectroscopy is a powerful tool in nuclear structure, but the interpretation of the intrinsic properties of the nucleus becomes complex if the parity of the emitted radiation is not known. This study aimed at developing a technique to measure linear polarization of γ rays emitted from non-oriented nuclear states for the clover detectors of the iThemba LABS AFRODITE array. Orientation was created by gating on a γ ray detected in one clover detector while measuring the polarization of another γ ray detected in coincidence in another detector. The clover detectors used in this study comprise four Ge crystals which were used as Compton polarimeters. It allowed us to measure linear polarization by studying the horizontal and vertical Compton scattering of γ rays in the four Ge crystals and measuring the polarization anisotropy, $A_p(E_\gamma)$. Theoretical values for all observed γ rays emitted in the β decay of ^{152}Eu were derived and compared with the measured polarization anisotropy to deduce the polarization sensitivity, $Q(E_\gamma)$. Furthermore, the degree of linear polarization was determined experimentally for the γ rays observed in ^{196}Hg populated in the β decay of ^{196}Tl using the deduced polarization sensitivity $Q(E_\gamma)$. The developed linear polarization technique was validated, and can be used in future experiments that intend to measure γ rays whose parities as well as mixing ratios are unknown.

1 Introduction

Gamma rays are usually emitted for each nuclear transition from an initial excited state to a state with lower excitation energy. There is a relationship between the properties of the nuclear states and the properties of the emitted γ rays which ensures that the conservation laws are properly obeyed. For instance, the energy of the emitted γ ray is equal to the difference between the energies of the initial and final nuclear states and so is the angular momentum and its projection along the axis of quantization [1]. Therefore, information on the properties of the nuclear states and even the structure of the nucleus of interest can be obtained by measuring the properties of the emitted γ ray such as its angular momentum and parity.

For nuclei produced in fusion-evaporation reactions, the angular momentum carried away by the emitted γ ray is often determined by carrying out angular distribution measurements. Such measurements give information on how the intensity of the emitted γ ray is distributed in space. But to determine the nature of the emitted γ ray as to whether it is electric or magnetic in nature, additional measurements such as linear polarization are needed [1]. Angular distribution and linear polarization measurements are necessary for the assignment of spins and parities and also for determining the mixing ratios of γ rays with mixed multipole nature. However, unlike in nuclear reactions where the nuclei are oriented or aligned (their angular momenta are aligned perpendicular to the beam direction), in β decays the daughter nuclei are produced with nuclear states which are not oriented at all. This lack of orientation makes the angular distribution of the γ rays isotropic, thus it is impossible to measure the angular momenta carried away by the emitted γ rays. This also makes it impossible to distinguish whether the emitted γ ray is of electric or magnetic nature. In this work, we aimed to develop a technique to measure the linear polarization of γ rays emitted from non-oriented nuclear states using the clover detectors of the iThemba LABS AFRODITE array [2, 3]. This includes measuring the polarization sensitivity factor $Q(E_\gamma)$ for these clover detectors using a ^{152}Eu radioactive source and also testing the technique with in-beam data where non-oriented nuclear states were produced in β decay.

2 Theoretical Background

2.1 Angular Correlation

Among the various methods of creating nuclear orientation, the most often used method for nuclear states produced in β decay is angular correlation. Angular correlation involves observing a sequential γ-ray emission. Orientation is created by choosing the emission of the first γ ray in a fixed direction while the second γ ray is detected in a direction different from that of the first. In practice, this corresponds to an event where two detectors positioned at an angle θ with respect to each other fire in coincidence. Thus, we select data where the first γ ray is detected in detector one, the second γ ray is observed in detector two, within a specified coincidence window. The angular correlation function W(θ) describes the probability of finding the second γ ray (γ_2) at an angle θ with respect to the direction of emission of the first γ ray (γ_1). The angular correlation function is given as:

$$W(\theta) = \sum_{n=0,2,4} a_n P_n(\cos\theta), \qquad (1)$$

where a_n are the corresponding angular correlation coefficients, $a_n = A_n(\gamma_1)A_n(\gamma_2)$, with $A_n(\gamma_1)$ and $A_n(\gamma_2)$ being the angular distribution coefficient for the first and second emitted γ ray, respectively, while the $P_n(\cos\theta)$ are Legendre polynomials [4].

2.2 Linear Polarization

The direction of emission of the two coincidence γ rays form a coincidence plane, see figure 1. The polarization of the γ ray refers to the orientation of the electric field vector with respect to the coincidence plane determined by the angle ψ in figure 1. Measuring the linear polarization of the first γ ray requires setting a coincidence gate on the second γ ray. Depending on the orientation of the electric field of the γ ray with respect to the coincidence plane the probability for the emitted γ ray to undergo Compton scattering in a vertical or horizontal direction differs. For instance, when the electric field vector is perpendicular to the coincidence plane horizontal Compton scattering, (in detectors D_1-D_3), is more probable, while vertical, (in detectors D_1-D_4) Compton scattering is more likely when the electric field is parallel to the coincidence plane.

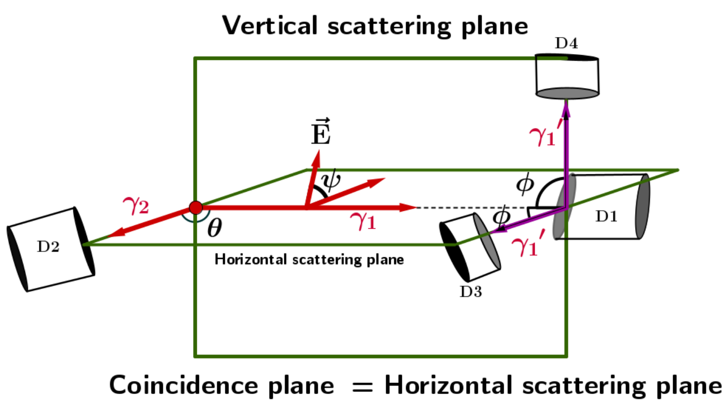

Figure 1: Illustration of a typical linear polarization set up, where the polarization of γ_1 is measured in coincidence with γ_2. The polarization (determining the orientation of the electric field vector $\vec{E}$) is deduced by measuring the number of Compton scattered, (in detectors D_1-D_3 and in detectors D_1-D_4) events. Compton scattering planes are defined by the direction of γ_1 and the direction of the scattered γ ray, $\gamma_1{}'$.

The directional polarization correlation describes the intensity of γ_1 depending on the angle θ with respect to γ_2 and the orientation of the electric field vector $\vec{E}$ with respect to the coincidence plane and is given as:

$$\mathrm{W}(\theta,\psi) = \sum_{\mathrm{n}=2,4} \mathrm{a_n}'\mathrm{P_n}^{(2)}(\cos\theta)\cos(2\psi), \tag{2}$$

where $\mathrm{P}_{\nu}^{(2)}(\cos\theta)$ are the associated Legendre polynomials [5]. The coefficients $a_n{}'$ are :

$$a_n{}' = A_n(\gamma_1)(-1)^{\sigma} f_n(LL')A_n(\gamma_2), \tag{3}$$

where σ and $f_n(LL')$ refer to the transition for which we measure polarization, γ_1 in figure 1, and, $\sigma = 0$ for electric and $\sigma = 1$ for magnetic radiation. The values of L and L' refer to the multipole order, and for a pure transition $L' = L$ and for a mixed one, $L' = L + 1$. The values of $f_n(LL')$ are given in [5].

Linear polarization of the emitted γ ray is defined in terms of the probabilities for the electric field vector to be parallel to the coincidence plane, W (θ, ψ = 0°), and to be perpendicular to the coincidence plane, W (θ, ψ = 90°), as:

$$\mathrm{P}(\theta) = \frac{\mathrm{W}(\theta,\psi=0^\circ) - \mathrm{W}(\theta,\psi=90^\circ)}{\mathrm{W}(\theta,\psi=0^\circ) + \mathrm{W}(\theta,\psi=90^\circ)}. \tag{4}$$

2.3 Polarization anisotropy

The expression for the number of vertically and horizontally Compton scattered events is proportional to the differential cross section as given by the Klein-Nishina formula [6]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{r}_0^2}{2}\left(\frac{\mathrm{E}'_\gamma}{\mathrm{E}_\gamma}\right)^2\left(\frac{\mathrm{E}'_\gamma}{\mathrm{E}_\gamma} + \frac{\mathrm{E}_\gamma}{\mathrm{E}'_\gamma} - 2\sin^2\phi\cos^2\varepsilon\right), \tag{5}$$

where r_0 is the classical electron radius given as $r_0 = \frac{e}{m_e c^2}$ with $m_e c^2 = 511$ keV the rest energy of the electron, ε is the angle between the electric field vector of the emitted γ ray and the Compton scattering plane, whereas ϕ is the Compton scattering angle between γ_1 and $\gamma_1{}'$, see figure 1. For horizontal and vertical Compton scattering, we have $\psi = \varepsilon$ and $\varepsilon = 90 - \psi$, respectively. The experimentally measured horizontally, N_h, and vertically, N_v, scattered events are,

$$\mathrm{N_h} = \mathrm{W}(\theta,\psi=0^\circ)\frac{\mathrm{d}\sigma(\phi,\varepsilon=0^\circ)}{\mathrm{d}\Omega} + \mathrm{W}(\theta,\psi=90^\circ)\frac{\mathrm{d}\sigma(\phi,\varepsilon=90^\circ)}{\mathrm{d}\Omega}, \tag{6}$$

$$\mathrm{N_v} = \mathrm{W}(\theta,\psi=0^\circ)\frac{\mathrm{d}\sigma(\phi,\varepsilon=90^\circ)}{\mathrm{d}\Omega} + \mathrm{W}(\theta,\psi=90^\circ)\frac{\mathrm{d}\sigma(\phi,\varepsilon=0^\circ)}{\mathrm{d}\Omega}. \tag{7}$$

Experimentally we measure the polarization anisotropy, $A_p(E_\gamma)$ which is:

$$A_p(E_\gamma) = \frac{aN_v - N_h}{aN_v + N_h}, \tag{8}$$

where $a = \frac{N_h}{N_v}$ is the relative efficiency and is obtained from direct spectra of horizontally and vertically scattered γ rays [7]. The polarization sensitivity $Q(E_\gamma)_{\mathrm{exp}}$ reflects the efficiency of the set-up for linear polarization and is obtained by comparing the polarization anisotropy, $A_p(E_\gamma)$, and the theoretical linear polarization, P(θ), for a cascade of transitions linking three states with known spins and parities. It is expressed as:

$$\mathrm{Q(E_\gamma)_{exp}} = \frac{\mathrm{A_p(E_\gamma)}}{\mathrm{P}(\theta)}. \tag{9}$$

3 Experimental Setup and Data Analysis

The data analysed in this study was collected using the iThemba LABS AFRODITE array [2, 3]. This setup consisted of 8 clover detectors positioned at 90° and 135° with respect to the beam direction. Linear polarization depends on the angle θ defined by the directions of the γ rays emitted in coincidence which in our case was the angle between two clover detectors in the same plane. Thus, pairs for the clover detectors were formed defining angles θ = 45°, 90°, and 135°. Since polarization is symmetric around 90°, the corresponding symmetric detectors angles were summed resulting in $\theta = 45°$ and 90° for linear polarization measurements. It is also important to mention that the data collected was a run with a ^{152}Eu radioactive source for 12 hours which was meant for efficiency calibration purpose. In-beam experimental data were also collected following a fusion-evaporation reaction involving an ^{18}O beam and ^{181}Ta target from which depending on the number of neutrons evaporated, thallium nuclei with mass numbers 194, 195, and 196 were formed. However, within these data some γ-ray transitions belonging to ^{196}Hg produced following the β decay of ^{196}Tl were observed. The data showed good statistics for linear polarization measurements, see the spectra gated on the 636-keV γ ray in figure 2. The observed γ rays with energies of 426, 636, 735 and 748 keV link low-spin states in ^{196}Hg populated below $I = 6^{+}$ [8].

To measure the polarization anisotropy, data were sorted into vertical and horizontal polarization matrices comprising events where two coincident gamma rays were detected, one of which had Compton-scattered in horizontal or vertical direction with respect to the coincidence plane. The sorted matrices were then converted into RADWARE [9] format. Gated spectra were created to reveal coincidence counts in the corresponding γ ray peaks. The gated spectra consist of all γ rays emitted in coincidence with the γ ray peak on which the gate is set, see gated spectra in figure 2. For each gated spectrum corresponding background was created by gating on a background region next to the peak of interest.

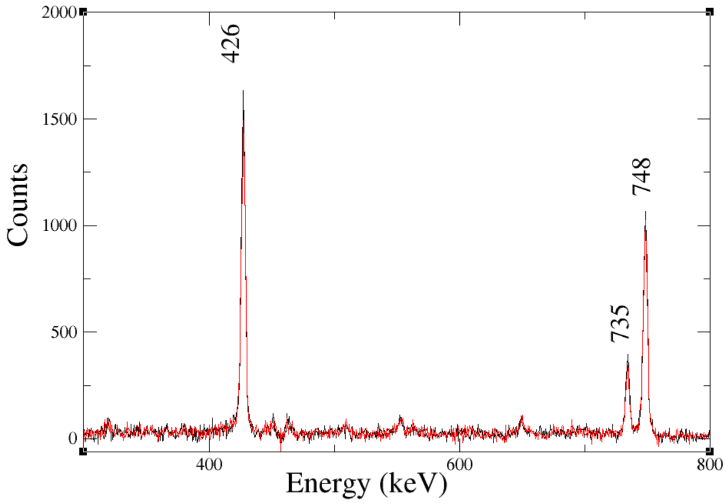

Figure 2: Spectra gated on the 636-keV transition of ^{196}Hg shown in black for vertical scattering and in red for horizontal scattering.

Linear polarization depends on the Compton scattering of the emitted γ ray between the adjacent detectors. The intensity of the vertically and horizontally Compton-scattered events between the adjacent crystals were obtained by fitting the γ-ray peaks of interest in the background-subtracted gated spectra to measure the polarization anisotropy, $A_p(E_\gamma)$ as per eq. 8. The relative efficiency, a was deduced using direct polarization spectra. The polarization anisotropy, $A_p(E_\gamma)$, was then compared with the theoretical linear polarization, $P(\theta)$, to deduced the polarization sensitivity, $Q(E_\gamma)_{\text{exp}}$ using eq. 9.

4 Results and Discussion

4.1 Polarization Sensitivity $Q(E_\gamma)$

The measured values of $A_p(E_\gamma)$ and $Q(E_\gamma)_{\text{exp}}$ are listed in table 1 for $\theta = 45°$ and $\theta = 90°$. They are also plotted in figure 3. For the 344-keV transition these values were measured by gating on two different peaks, 411 and 778 keV.

Table 1: Measured polarization anisotropy, $A_p(E_\gamma)$, and sensitivity, $Q(E_\gamma)_{\text{exp}}$, for the γ rays emitted from ^{152}Eu.

					$\theta = 90°$		$\theta = 45°$	
Gate	**Coincidence(keV)**	**L**	δ	a	$A_p(E_\gamma)$	$Q(E_\gamma)_{\textbf{exp}}$	$A_p(E_\gamma)$	$Q(E_\gamma)_{\textbf{exp}}$
g122	245	E2	0	0.962(4)	0.042(19)	0.25(11)	0.003(12)	0.04(18)
g778	344	E2	0	0.964(2)	-0.009(11)	0.08(11)	-0.007(8)	0.13(14)
g411	344	E2	0	0.964(2)	0.037(23)	0.22(14)	0.016(15)	0.23(22)
g344	411	E2	0	0.977(6)	0.037(21)	0.22(13)	0.005(14)	0.08(21)
g344	778	E1	0	0.975(2)	0.022(8)	0.21(8)	0.006(6)	0.11(11)
g245	867	M1+E2	$-6.5(3)$	0.979(4)	-0.029(16)	0.17(9)	0.002(10)	0.01(9)
g122	964	M1+E2	$-9.3(6)$	0.985(2)	0.007(10)	0.14(20)	-0.021(7)	0.06(2)
g122	1112	M1+E2	$-8.7(3)$	0.981(3)	-0.008(10)	0.03(4)	-0.002(7)	0.03(11)
g122	1408	E1	0	0.986(2)	-0.019(9)	0.04(2)	-0.007(6)	0.04(3)

The deduced polarization sensitivity, $Q(E_\gamma)_{\text{exp}}$ was fitted with the expression:

$$Q(E_\gamma) = Q_{pt}\big(E_\gamma \cdot b_1 + b_0\big), \tag{10}$$

where b_0 and b_1 are the fitting parameters [10]. Values of $b_1 = 0.00009(9)$ and b_0 = 0.23(6), see figure 3 were deduced.

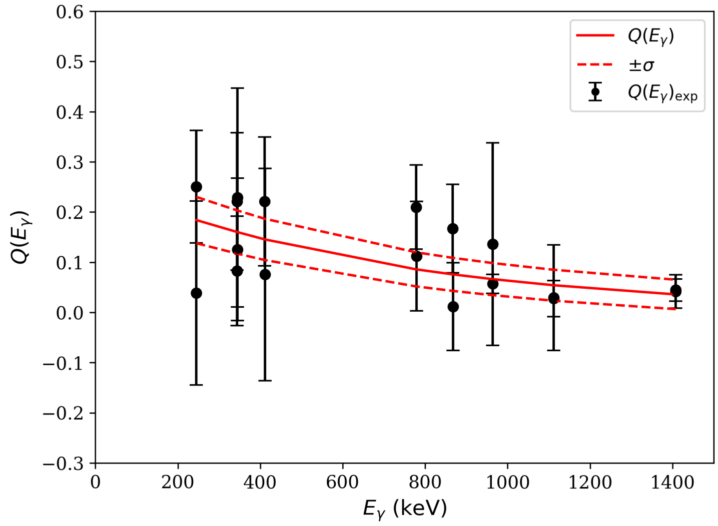

Figure 3: Polarization sensitivity results obtained for the clover detectors of the AFRODITE array (black) and the fitted with an expression eq 10 (red) with $b_1 = 0.00009(9)$ and $b_0 = 0.23(6)$.

The large uncertainty in the experimental data is due to low statistics, especially at low energy where Compton scattering not dominant.

4.2 *Testing the technique with ^{196}Hg*

Since the nuclear states resulting from β decay are non-oriented it is possible to test the developed linear polarization technique using the γ rays emitted from non-oriented nuclear states obtained in an in-beam experiment. The degree of linear polarization for the transitions in ^{196}Hg was established experimentally by using the determined AFRODITE polarization sensitivity $Q(E_\gamma)$, see figure 3, and by measuring the polarization anisotropies, $A_p(E_\gamma)$, and using eq 9. The results are given in table 2.

Table 2: Measured polarization anisotropy, $A_p(E_\gamma)$, and linear polarization $P(\theta)_{\text{exp}}$ for the γ rays of ^{196}Hg using the clover detector pairs at $\theta = 45°$ and $\theta = 90°$. The theoretical values of $P(\theta)$ are also listed for comparison.

				$\theta = 45°$			$\theta = 90°$		
Position in fig 4	**Gate**	E_γ**(keV)**	**L**	$A_p(E_\gamma)$	**P**$(\theta)_{\text{exp}}$	**P**(θ)	$A_p(E_\gamma)$	**P**$(\theta)_{\text{exp}}$	**P**(θ)
1	g636	426	E2	0.020(21)	0.14(15)	0.068	0.037(21)	0.25(16)	0.167
2	g748	636	E2	-0.004(13)	-0.040(12)	-0.055	-0.017(31)	-0.16(29)	-0.103
3	g735	636	E2	0.007(29)	0.06(26)	0.068	0.025(36)	0.23(34)	0.167
4	g426	636	E2	0.009(13)	0.088(12)	0.068	0.017(24)	0.16(23)	0.167
5	g636	735	E2	0.028(26)	0.29(29)	0.068	0.004(21)	0.05(22)	0.167
6	g636	748	E1	0.001(16)	0.01(18)	0.05	0.005(23)	0.06(25)	0.103

The results for the 636-keV γ ray were obtained from three different coincident gates, two of which form a quadrupole-quadrupole (E2-E2) cascades with the 636-keV transition and hence they have the same theoretical value for the corresponding linear polarization and the third forms a dipole-quadrupole (E1-E2) cascade. The values for the experimental and theoretical polarization show very good agreement for all measured transitions in ^{196}Hg as is evident in figure 4. Therefore, the technique works for all γ rays emitted from non-oriented nuclear states and as such, it can be applied to transitions with unknown parities.

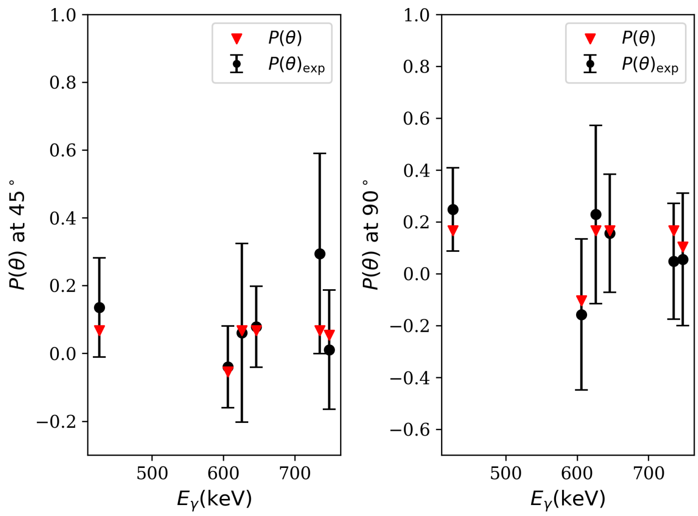

Figure 4: A comparison between the measured linear polarization $P(\theta)_{\text{exp}}$ (in black) and the corresponding theoretical value (in red) for the clover detector pairs at $\theta = 45°$ and $\theta = 90°$. The measurements are done for the transitions of ^{196}Hg. The three data points for the 636-keV γ ray are shifted a bit in energy, for better visibility.

5 Summary

This study describes a technique to measure the linear polarization of γ rays emitted from non-oriented nuclear states. The technique has been developed for the clover detectors of the iThemba LABS AFRODITE array and the AFRODITE polarization sensitivity, $Q(E_\gamma)$, has been deduced. In addition, the technique has been tested by measuring the degree of linear polarization for the γ rays of ^{196}Hg observed in the β decay of ^{196}Tl produced in an in-beam experiment. The degree of linear polarization obtained experimentally for the transitions in ^{196}Hg are found in very good agreement with the corresponding theoretical values, which confirms the validity of the technique. Thus, the developed technique can be applied for future studies that intend to assign parities to γ rays with unknown multipolarities and will also be crucial in determining the mixing ratios for those with mixed multipole nature.

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