Understanding Interacting Dark Energy from a Dynamical Systems Analysis Approach

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Abstract. Cosmological models in which dark matter and dark energy interact in a non-gravitational manner are known as Interacting Dark Energy (IDE) models and have been proposed to address many long-standing shortcomings and tensions in standard cosmology. Furthermore, recent results from the DESI Collaboration have suggested hints of dynamical dark energy, for which IDE models could provide a viable explanation. The relevance of IDE models underscores the need to understand their parameter space and potential limitations. Unfortunately, obtaining analytical solutions for the evolution of dark matter and dark energy densities is often not possible. To this end, we apply dynamical system analysis techniques to a well-known IDE model $Q = 3H\delta\rho_{\rm de}$, establishing an alternative method to find constraints on the parameter space that avoid the common pitfalls of these models, such as negative energy densities and future singularities, without the need for analytical solutions. We also show that our method can provide insight into the requirement for radiation, matter and dark energy dominated eras, accelerated future expansion, and how these models may address the coincidence problem. Using the techniques established here, researchers may investigate other IDE models that have escaped analysis to date.

1 Introduction

In standard cosmology, the expansion of the universe is described by the ΛCDM model, where the universe is dominated by two mysterious dark components, cold dark matter (CDM) and dark energy Λ, estimated to make up approximately 95% of the energy of the universe. Dark matter (DM) is usually modelled as a particle beyond the Standard Model introduced to explain galaxy rotation curves and many other astronomical and cosmological observations. Dark energy (DE) is usually assumed to be a cosmological constant Λ, which was introduced to account for the observed accelerated expansion of the universe. The ΛCDM model has been incredibly successful in describing cosmological data, as illustrated by measurements of the power spectrum of the Cosmic Microwave Background [1]. Although successful, the ΛCDM models face many challenges, which motivate research beyond the standard cosmological model. We will briefly mention five of these shortcomings, two of theoretical origin, and three that relate to recent increases in precision of cosmological data.

1. **The cosmological constant problem:** The predicted energy density of a cosmological constant is measured to be approximately 120 orders of magnitude smaller than the predicted value [2].

2. **The coincidence problem:** The DM and DE densities coincidentally have the same order of magnitude at the present moment when we measure it, while their energy densities are predicted to differ with many orders of magnitude in both the past and predicted future [3].

3. **The Hubble tension:** The $4\sigma - 6\sigma$ discrepancy of the estimation of the present expansion rate H_0 from early time probes such as CMB, and late time probes such as Type Ia Supernova [4].

4. **The $S8$ discrepancy:** The $2\sigma - 5\sigma$ discrepancy between late-time and early-time measurements of the parameter S_8, which is related to how matter clumps on cosmological scales [4].

5. **Hints of dynamical dark energy**: Recent measurements of baryonic acoustic oscillations (BAO) from the DESI collaboration provide a $2.8\sigma - 4.2\sigma$ preference for dynamical DE over the ΛCDM model [5].

In order to address the above-mentioned problems, a plethora of extensions to standard cosmology have been suggested, but we will focus on a group of models called interacting dark energy (IDE). IDE models are characterised by an hypothesised non-gravitational energy exchange between DM and DE. This has the consequence that the energies of neither DM nor DE are separately conserved, but instead the energy of the total dark sector is conserved, which leads to a modification of the standard conservation equation:

$$\begin{aligned} \dot{\rho}_{\text{dm}} + 3H\rho_{\text{dm}} = Q \quad &; \quad \dot{\rho}_{\text{de}} + 3H(1+\omega)\rho_{\text{de}} = -Q \\ \omega_{\text{dm}}^{\text{eff}} = -\frac{Q}{3H\rho_{\text{dm}}} \quad &; \quad \omega_{\text{de}}^{\text{eff}} = \omega_{\text{de}} + \frac{Q}{3H\rho_{\text{de}}}. \end{aligned} \tag{1}$$

In (1), ρ_{dm} and ρ_{de} are the energy densities of DM and DE, respectively, $\dot{\rho}$ denotes differentiation with respect to cosmic time, $H = \frac{\dot{a}}{a}$ is the Hubble parameter, a the scale factor and ω is the DE equation of state. Most important is Q, the interaction function whose sign determines the direction of energy transfer, such that energy flows from DE to DM if $Q > 0$ (iDEDM regime), while energy flows from DM to DE if $Q < 0$ (iDMDE regime). The effect of the interaction can also be encapsulated using effective equations of state $\omega_{\text{dm/de}}^{\text{eff}}$, which provide equivalent fluid descriptions without an interaction, but instead with dynamical equations of state. IDE models were initially introduced to address the coincidence problem, as the interaction can allow DM and DE to dilute with expansion at the same rate [3], such that the problem is solved for either the past or future if $\omega_{\text{dm}}^{\text{eff}} = \omega_{\text{de}}^{\text{eff}}$. In recent years, it has been shown that IDE models can also help alleviate H_0 and S_8 tensions [4], increasing their popularity. Lastly, IDE models provide a natural mechanism to account for the recent hints of dynamical dark energy, while also allowing for phantom crossings [5]. Although IDE models have become popular, we would like to highlight the shortcomings of IDE models, such as negative energy densities and future singularities, and to provide find conditions to avoid them, if possible.

One of the problems is that the function Q is not agreed upon, and is usually decided phenomenologically. This leads to a plethora of interaction kernels Q, very few of which (1) can be analytically solved. Without analytical solutions for the evolution of ρ_{dm} and ρ_{de}, the background dynamics of these models can be hard to predict, especially with regards to the possible presence of both negative energy densities and big rip future singularities. To this end, we apply techniques from dynamical systems analysis to find conditions that ensure radiation, matter and DE dominated eras, positive energy densities, future accelerated expansion, avoid a future singularity and to address the coincidence problem, all without the need for analytical solutions. We have chosen the most popular interaction kernel $Q = 3H\delta\rho_{\text{de}}$ to show that our method finds convergent results with those obtained from analytical solutions given in [6]. The methods established here can be used to obtain similar conditions for any interaction kernel Q.

2 Background equations

To address the coincidence problem, we require that DM and DE redshift and dilute at the same rate in either the distant past or future. The ratio of DM to DE is given by:

$$r = \frac{\rho_{\text{dm}}}{\rho_{\text{de}}} = \frac{\Omega_{\text{dm}}}{\Omega_{\text{de}}} = \frac{\rho_{\text{dm},0}a^{-3(1+\omega_{\text{dm}}^{\text{eff}})}}{\rho_{\text{de},0}a^{-3(1+\omega_{\text{de}}^{\text{eff}})}} = r_0 a^{-3(\omega_{\text{dm}}^{\text{eff}} - \omega_{\text{de}}^{\text{eff}})} \quad ; \quad \zeta = 3(\omega_{\text{dm}}^{\text{eff}} - \omega_{\text{de}}^{\text{eff}}). \tag{2}$$

The magnitude of the deviation from $\zeta = 0$ indicates the degree of the coincidence problem, as $\zeta = 0$ corresponds to $\omega_{\text{dm}}^{\text{eff}} = \omega_{\text{de}}^{\text{eff}}$, where DM and DE redshifts at the same rate, fixing their ratio $r = \text{constant}$. In the ΛCDM model we have $\zeta = 3$, so if $\zeta < 3$ the coincidence problem is alleviated, which happens in the iDEDM regime, while $\zeta > 3$ worsens the problem, which happens in the iDMDE regime [3, 6]. Besides DM and DE, the universe also contains radiation (r) and baryonic matter (bm), whose density evolves with expansion of the universe as $\rho_{\text{r}} = \rho_{(\text{r},0)}a^{-4}$ and $\rho_{\text{bm}} = \rho_{(\text{bm},0)}a^{-3}$, respectively. The entire model may also be approximated as a single fluid, which has a total effective equation of state $\omega_{\text{tot}}^{\text{eff}}$ given by:

$$\omega_{\text{tot}}^{\text{eff}} = \frac{P_{\text{tot}}}{\rho_{\text{tot}}} = \frac{1}{3}\Omega_{\text{r}} + \omega\Omega_{\text{de}}, \tag{3}$$

where the last equality is obtained by assuming a flat universe, such that the density parameters $\Omega_i = \frac{8\pi G}{3H^2}\rho_i$ add up to satisfy $\Omega_{\text{r}} + \Omega_{\text{bm}} + \Omega_{\text{dm}} + \Omega_{\text{de}} = 1$. What makes $\omega_{\text{tot}}^{\text{eff}}$ useful is that it determines the fate of the universe, such that $\omega_{\rm tot}^{\rm eff} \leq -\frac{1}{3}$ corresponds to accelerated expansion, while $\omega_{\rm tot}^{\rm eff} < -1$ corresponds to a big rip future singularity. A big rip singularity is characterised by both the DE density and the scale factor becoming infinite ($\rho_{\rm de} \to \infty$; $a \to \infty$) within a finite time interval. This behaviour can be considered non-physical by researchers, and models that avoid this are preferred.

3 Dynamical systems analysis

Dynamical systems analysis is a well-established tool to study the stability of a system of equations. Solutions to the system of equations are known as critical points, where many of the trajectories corresponding to functions with different initial conditions may either diverge or converge. If the system is perturbed around a critical point, the system may diverge away from the point, called an unstable node, source or past attractor. Conversely, if the trajectories instead converge at the critical point, the point is classified as a stable node, sink or future attractor. Lastly, if some trajectories converge, while others diverge, the critical point is known as a saddle point. Mathematically, these points are classified according to sign of the eigenvalues obtained from the Jacobian matrix of the system of equations at that point. Specifically, if all eigenvalues are positive, the point is an unstable node; if some eigenvalues are positive while others are negative, we have a saddle point; and if all eigenvalues are negative we have a stable node. Additionally, zero eigenvalues correspond to a manifold of solutions. See [7] for applications of these techniques to cosmology. The behaviour of the system at the future attractor is especially useful in determining the fate of the universe, no matter the initial conditions.

We will now consider the dynamical behaviour of an IDE model with the interaction kernel $Q = 3\delta H\rho_{\rm de}$. The system of equations was derived in [6], but we have added the flatness assumption to reduce the number of equations ($\Omega_{\rm r} = 1 - \Omega_{\rm bm} - \Omega_{\rm dm} - \Omega_{\rm de}$), as well as taken the derivative to the Hubble parameter $\Omega_{\rm i}' = \frac{d}{d\zeta}\Omega_{\rm i}$, which leads to the dynamical system:

$$\begin{aligned}
\Omega_{\rm de}' &= \Omega_{\rm de}\left[1 - \Omega_{\rm bm} - \Omega_{\rm dm} - \Omega_{\rm de}\left(1 - 3\omega\right) - 3\omega\right] - 3\delta\Omega_{\rm de} \\
\Omega_{\rm dm}' &= \Omega_{\rm dm}\left[1 - \Omega_{\rm bm} - \Omega_{\rm dm} - \Omega_{\rm de}\left(1 - 3\omega\right)\right] + 3\delta\Omega_{\rm de}, \\
\Omega_{\rm bm}' &= \Omega_{\rm bm}\left[1 - \Omega_{\rm bm} - \Omega_{\rm dm} - \Omega_{\rm de}\left(1 - 3\omega\right)\right],
\end{aligned} \tag{4}$$

where we also used the relation that $\frac{8\pi G}{3H^2}\rho_i = \Omega_i$. From the dynamical system in (4), we obtained three solutions corresponding to three critical points, alongside the eigenvalues λ of the Jacobian matrix at those points, whose stability we analyse one at a time. For this study, we have two assumptions regarding the parameter space: 1. $\omega < 0$ (DE has negative pressure) ; 2. $\delta < |\omega|$ (the interaction strength is not too strong)

Critical Point $P_{\rm r}$: radiation-dominated phase.

$$\Omega_{\rm bm} = 0, \quad \Omega_{\rm dm} = 0, \quad \Omega_{\rm de} = 0, \quad \to \quad \Omega_{\rm r} = 1 \quad ; \quad \lambda = \begin{bmatrix} 1 - 3(\delta + \omega) \\ 1 \\ 1 \end{bmatrix}. \tag{5}$$

Since we have the assumed condition that $(\delta + \omega) < 0$, this implies that the first eigenvalue is positive $1 - 3(\delta + \omega) > 0$. Since all eigenvalues are positive, this radiation-dominated phase is an unstable node (source).

$$\underline{\text{Conditions for}} : \text{Radiation-dominated unstable node (source)} \; \left\{ (\delta + \omega) < \tfrac{1}{3} \right. . \tag{6}$$

Critical Point $P_{\rm m}$: matter-dominated phase.

$$\Omega_{\rm bm} = -\Omega_{\rm dm} + 1, \quad \Omega_{\rm dm} = \Omega_{\rm dm}, \quad \Omega_{\rm de} = 0, \quad \to \quad \Omega_{\rm r} = 0 \quad ; \quad \lambda = \begin{bmatrix} 0 \\ -1 \\ -3(\delta + \omega) \end{bmatrix}. \tag{7}$$

The coordinates in (7) correspond to a combination of baryonic and DM domination, alternatively, a *matter-dominated phase*, as shown by $\Omega_{\rm m} = \Omega_{\rm bm} + \Omega_{\rm dm} = -\Omega_{\rm dm} + 1 + \Omega_{\rm dm} = 1$. This critical point is also not a single point, but a line on the axis where the combination of the two densities equal to one, as seen in Figure 1 below. This behaviour is illustrated by the first eigenvalue being zero, indicating a line or manifold that consists of a continuous set of equilibria where the sum of baryonic and DM is equal to one. Furthermore, the second eigenvalue is negative, while the third eigenvalue is positive, which implies that this manifold also acts as a saddle point.

$$\underline{\text{Conditions for}} : \text{Matter-dominated saddle manifold} \; \left\{ \delta < -\omega \right. . \tag{8}$$

Critical Point $P_{\mathrm{dm+de}}$: dark energy hybrid dominated phase.

$$\Omega_{\mathrm{bm}}=0, \quad \Omega_{\mathrm{dm}}=-\frac{\delta}{\omega}, \quad \Omega_{\mathrm{de}}=1+\frac{\delta}{\omega}, \quad \rightarrow \quad \Omega_{\mathrm{r}}=0 \quad ; \quad \lambda=\begin{bmatrix} 3(\delta+\omega)-1 \\ 3(\delta+\omega) \\ 3(\delta+\omega) \end{bmatrix}. \tag{9}$$

From the coordinates in (9) we see that $\Omega_{\mathrm{dm}}+\Omega_{\mathrm{de}}=1$ at this critical point, which corresponds to DM-DE hybrid dominated phase. We may note that DE will dominate DM at this critical point if $\delta<-\omega/2$, as this will cause $\Omega_{\mathrm{de}}>\Omega_{\mathrm{dm}}$. Immediately from the matter coordinate of the critical point, alongside the assumption that $\omega<0$, we note that DM will be negative if:

$$\underline{\text{Conditions for }}\ \Omega_{\mathrm{dm}}>0: \quad \omega<0 \text{ and } \delta>0 \text{ (iDEDM regime)} \tag{10}$$

For the DE to remain positive $\Omega_{\mathrm{de}}=1+\frac{\delta}{\omega}>0$ we need $\delta<-\omega$, which justifies our initial assumption about the magnitude of the interaction strength. From the assumption that $(\delta+\omega)<0$, this implies $3(\delta+\omega)<0$ and $3(\delta+\omega)-1<0$, which means both eigenvalues are negative and the critical point is a stable node (sink).

$$\underline{\text{Conditions for}}: \text{ Positive dark matter-dark energy hybrid dominated stable node } \begin{cases} \omega<0 \\ 0<\delta<-\omega \end{cases}. \tag{11}$$

The critical points and the behaviour of trajectories around these points can be seen by plotting phase portraits of the dynamical system in (4). In Figure 1, we see that this model has a past radiation-dominated attractor P_{r}, but does not show a single baryonic matter or DM-dominated saddle point. Instead, a manifold that consists of a continuous set of equilibria points can be seen located on the line where $\Omega_{\mathrm{bm}}=\Omega_{\mathrm{bm}}+\Omega_{\mathrm{dm}}=1$. This manifold acts as a continuous set of matter-dominated saddle points P_{m}. To more clearly see the effects of the interaction term on the dark components, we also consider the 2D projection in the $(\Omega_{\mathrm{dm}},\Omega_{\mathrm{de}})$ plane in the bottom panels of Figure 1, where we set $\Omega_{\mathrm{bm}}=0$, as baryonic matter does not interact and will not influence if DM or DE crosses into negative densities. We see that the interaction most noticeably changes the stable node which acts as a future attractor from complete DE dominance in ΛCDM to a DM-DE hybrid dominance. Importantly, the sign of the interaction constant δ, which determines the direction of energy flow, will determine whether this future attractor point has negative energies. *For a small interaction in the iDEDM regime ($\delta>0$), all energies will remain positive, while in the iDMDE regime ($\delta<0$), DM will always become negative in the future ($\Omega_{\mathrm{dm}}<0$).*

We now need to consider where the trajectories become negative by doing a boundary analysis. For the DE density to remain positive we need the DE flow lines $\Omega'_{\mathrm{de}}\geq 0$ at the boundary $\Omega_{\mathrm{de}}=0$, so that the flow cannot cross into negative Ω_{de}. When we substitute the condition $\Omega_{\mathrm{de}}=0$ into Ω'_{de} found in the dynamical system (4), we obtain $\Omega'_{\mathrm{de}}(\Omega_{\mathrm{de}}=0)=0$. Thus, we see that Ω'_{de} vanishes here and the flow is tangent to the line, which implies that the trajectories cannot cross into negative Ω_{de}, ensuring positive DE densities for any choice of parameters in the physically viable area. This is a direct consequence of the fact that as $\rho_{\mathrm{de}}\rightarrow 0$, then $Q=3H\rho_{\mathrm{de}}\rightarrow 0$ and the energy flow stops before negative DE can be reached.

For DM to remain positive, we need the trajectories to remain within region bounded by three invariant sub-manifolds, illustrated by the green triangle in bottom left panel of Figure 1, connecting the critical points P_{r}, P_{m} and $P_{\mathrm{dm+de}}$. No trajectories will cross the line connecting P_{r} and P_{m}, as it is an invariant sub-manifold, while trajectories will also remain within the line connecting P_{m} and $P_{\mathrm{dm+de}}$, which is guaranteed from the flatness assumption $\Omega_{(\mathrm{dm},0)}+\Omega_{(\mathrm{de},0)}\leq 1$. Finally the line connecting P_{r} $(\Omega_{\mathrm{dm}},\Omega_{\mathrm{de}})=(0,0)$ and $P_{\mathrm{dm+de}}$ $(\Omega_{\mathrm{dm}},\Omega_{\mathrm{de}})=(-\frac{\delta}{\omega},1+\frac{\delta}{\omega})$ has the gradient $m=\frac{\Delta\Omega_{\mathrm{de}}}{\Delta\Omega_{\mathrm{dm}}}=\frac{1+\frac{\delta}{\omega}-0}{-\frac{\delta}{\omega}-0}=-\frac{\delta+\omega}{\delta}$. It should be noted that the slope will be positive $m>0$, from the assumption $\delta+\omega<0$ and $\delta>0$ from previously derived constraint in (11). The positive slope ensures the slope is in the positive DM domain ($\Omega_{\mathrm{dm}}>0$). We can find additional constraints on the initial coordinates $(\Omega_{(\mathrm{dm},0)},\Omega_{(\mathrm{de},0)})$ to ensure positive energy, by requiring that these coordinates are below the invariant line connecting P_{r} and $P_{\mathrm{dm+de}}$:

$$\Omega_{(\mathrm{de},0)}\leq-\frac{\delta+\omega}{\delta}\Omega_{(\mathrm{dm},0)} \quad \rightarrow \quad \delta\leq-\frac{\omega}{\left(1+\frac{1}{r_0}\right)}, \tag{12}$$

where $r_0=\frac{\Omega_{(\mathrm{dm},0)}}{\Omega_{(\mathrm{de},0)}}$ and $\delta>0$. Condition (12) can be combined with the constraint for positive critical points in (11) to obtain (1), which ensures positive energies at all points in the cosmological evolution. This is the same condition found in [6] using analytical solutions for ρ_{dm} and ρ_{de}. The asymptotic past and future behaviour of the model can be understood by substituting the critical points into (2) and (3), yielding Table 1.

$$\boxed{\text{Conditions for } \Omega_{\mathrm{dm/de}}\geq 0 \text{ throughout cosmological evolution: iDEDM with } 0\leq\delta\leq-\frac{\omega}{\left(1+\frac{1}{r_0}\right)}}. \tag{13}$$

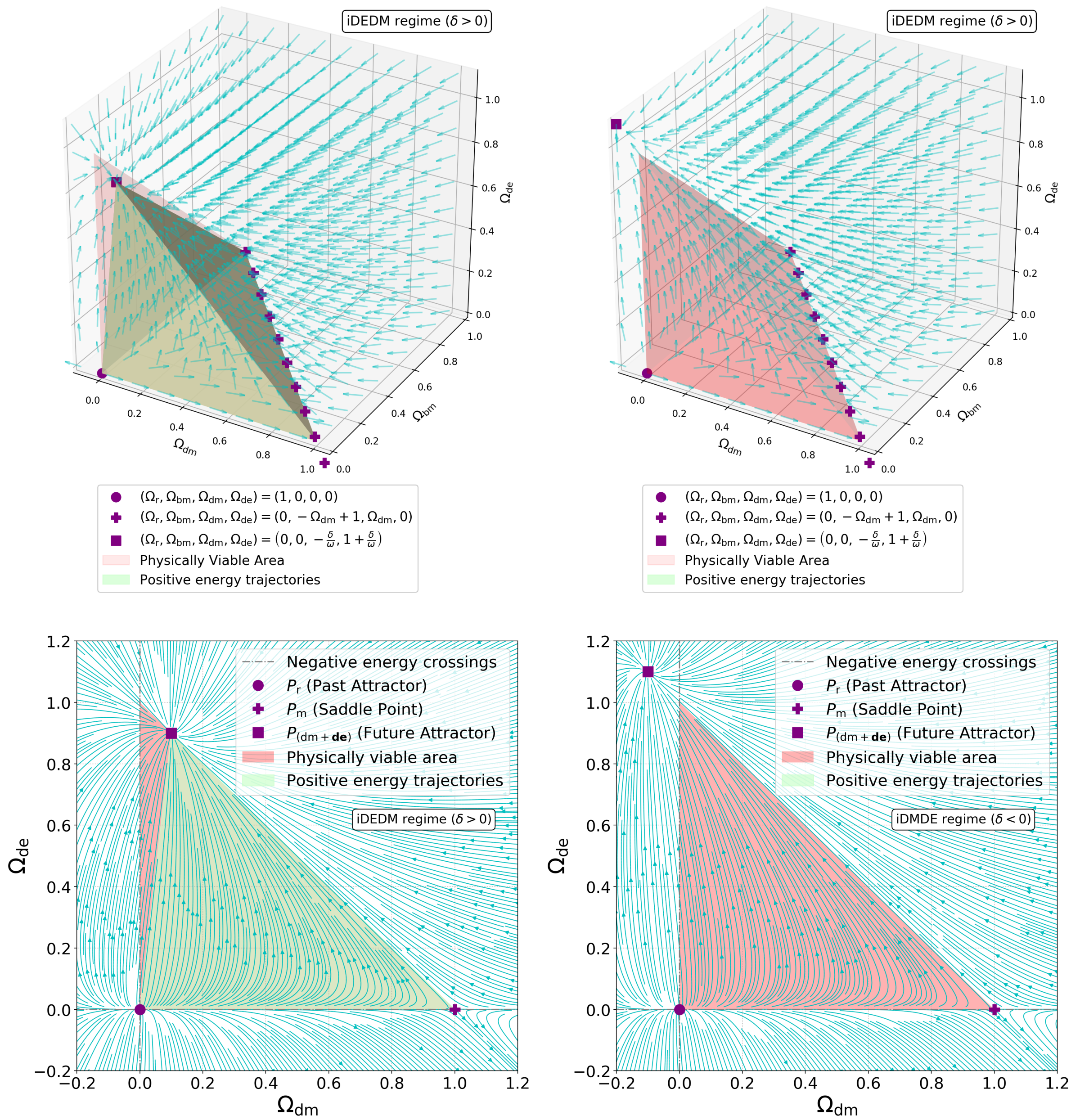

Figure 1: Phase portraits showing the evolution density parameters for the IDE model with interaction kernel $Q = 3H\delta\rho_{\rm de}$. The top panels show 3D phase portraits with positive energy trajectories only found in the iDEDM regime $\delta = +0.1$ (left panels) and only negative energy trajectories in the iDMDE regime $\delta = -0.1$ (right panels). All trajectories share a radiation-dominated past attractor $P_{\rm r}$, a matter-dominated saddle point $P_{\rm m}$ and a DM-DE hybrid dominated future attractor $P_{\rm dm+\mathbf{de}}$, where DM becomes negative in the iDMDE regime, indicated by trajectories leaving the red physically viable area. A 2D projection of the phase portraits in the $(\Omega_{\rm dm}, \Omega_{\rm de})$ plane (bottom panels) is obtained by setting $\Omega_{\rm bm} = 0$. The bounded region connecting the three critical points, indicated by the green triangle, is used to derive the positive energy conditions found in (13).

	P_{r}	P_{m}	$P_{\mathrm{dm+\mathbf{de}}}$
Class	Unstable node (source)	Saddle Point	Stable node (sink)
Ω_{r}	1	0	0
Ω_{bm}	0	$-\Omega_{\mathrm{dm}}+1$	0
Ω_{dm}	0	Ω_{dm}	$-\frac{\delta}{\omega}$
Ω_{de}	0	0	$1+\frac{\delta}{\omega}$
r	∞	∞	$-\frac{\delta}{\delta+\omega}$
$\omega_{\mathrm{dm}}^{\mathrm{eff}}$	0	0	$\omega+\delta$
$\omega_{\mathrm{de}}^{\mathrm{eff}}$	$\omega+\delta$	$\omega+\delta$	$\omega+\delta$
ζ	$-3(\omega+\delta)$	$-3(\omega+\delta)$	0
$\omega_{\mathrm{tot}}^{\mathrm{eff}}$	$\frac{1}{3}$	0	$\omega+\delta$

Table 1: Behaviour of IDE model $Q = 3H\delta\rho_{\mathrm{de}}$ at critical points.

4 Conclusions

In this study, we performed a dynamical systems analysis of the IDE model $Q = 3H\delta\rho_{\mathrm{de}}$, resulting in the positive energy conditions (13), which implies that *DM can only remain positive if there is a small interaction in the iDEDM regime*. This analysis also resulted in Table 1, where we see that our model is guaranteed to have past eras dominated by radiation P_{r} and matter P_{m}, during which the severity of the coincidence problem is given by $\zeta = -3(\delta+\omega)$. Since $\omega < 0$ and in the iDEDM regime $\delta > 0$, we have $\zeta < 3$, thus *alleviating* the coincidence problem in the past, while for the iDMDE regime where $\delta < 0$, *worsening* the coincidence problem.

At the future attractor $P_{\mathrm{dm+\mathbf{de}}}$, there is a hybrid DM-DE dominant phase , during which the DE and DM redshifts at the same rate $\omega_{\mathrm{dm}}^{\mathrm{eff}} = \omega_{\mathrm{de}}^{\mathrm{eff}} = \omega + \delta$, implying a fixed ratio of $r = -\frac{\delta}{\delta+\omega}$ and $\zeta = 0$, *solving the coincidence problem in the future*. At this final attractor point, the model will show late-time accelerated expansion as long as $\omega_{\mathrm{tot}}^{\mathrm{eff}} = \omega+\delta \leq -\frac{1}{3}$, which will hold as long as $\delta \leq -\omega - \frac{1}{3}$. Lastly, DE can be in the phantom regime $\omega < -1$, but avoid a big rip singularity in the iDEDM regime ($\delta > 0$), as long as the interaction strength is sufficiently positive such that $\omega_{\mathrm{tot}}^{\mathrm{eff}} = \omega + \delta \geq -1$. This is obtained when $\delta \geq -(\omega+1)$. All results obtained in this study agree with those in [6], establishing these dynamical systems analysis techniques as an alternative method to understand the parameter space of IDE models, even without analytical expressions for the evolution of the density of DM and DE. We recommend that other researchers apply similar analysis to IDE models, so that we may better understand the implications of models that are candidates to address cosmological tensions.

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