Testing $f(Q)$ gravity as a solution for the H_0 and S_8 tensions

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Abstract. In this work, we investigate a modified gravity framework based on the $f(Q)$ gravity model, where Q is the non-metricity scalar, specifically focussing on the parameterisation $f(Q) = \alpha + \beta Q^n$ as solutions for the Hubble constant (H_0) and the matter fluctuation amplitude parameter (S_8) tensions. Using recent observational data sets including baryon acoustic oscillations (BAO) from Dark Energy Spectroscopic Instrument (DESI), cosmic chronometers (CC), and type Ia supernovae from the Pantheon+ and SH0ES compilations, we constrain the free parameters of the Lambda Cold Dark Matter (ΛCDM) model and our model via the Markov Chain Monte Carlo (MCMC) method. Our analysis shows that the $f(Q)$ model can accommodate observational data with large error margins in the derived values of H_0 and reveals possible degeneracies when assuming the solution $f_Q \equiv \frac{\partial f}{\partial Q} = 1$ today. We compare our findings with previous studies that relax certain assumptions of the model and find improved parameter constraints. We outline plans for future work that will perform a comprehensive statistical assessment of the $f(Q)$ model's ability to resolve the H_0 and S_8 tensions by combining early- and late-time cosmological measurements without restrictive assumptions.

1 Introduction

In modern physics, the indispensable and foundational theory of space, time, and gravitation, where gravity is understood as the curvature of spacetime is *General Relativity* (GR), formulated by Einstein in 1915 [1, 2]. GR has proven remarkably successful, accurately predicting a wide range of phenomena, including planetary motion, light deflection, gravitational time delay, and black hole dynamics. It has been rigorously tested across a wide range of scales, from submillimeter laboratory experiments to solar system distances ($\sim 10^{14}\ m$), including strongly gravitating binary pulsar systems. A major milestone came with the first direct detection of gravitational waves by the Laser Interferometer Gravitational-Wave Observatory, coinciding with the 100th anniversary of GR [2]. However, several large-scale observations such as the accelerated expansion of the universe from type Ia supernovae (SNe Ia) observations [3], and galactic rotation curve velocity measurements [4, 5] pose significant challenges for GR.

To address these discrepancies within the standard cosmological framework, the Lambda Cold Dark Matter (ΛCDM) model was introduced, which incorporates a cosmological constant (Λ) and cold dark matter as essential components [6, 7, 8]. Although the ΛCDM model resolves some of the shortcomings of GR such as the observed accelerated expansion of the universe, it also introduces its own challenges. One such issue is the fine-tuning problem, where the theoretical value of the dark energy density derived from quantum field theory is $\gtrsim 10^{121}$ times the observed value. Another is the coincidence problem, where the dark energy and dark matter densities are of the same order of magnitude today, despite evolving differently over time. Furthermore, observational tensions exists between early- and late-time measurements of key cosmological parameters, namely the measured values of the

Hubble constant (H_0) and the matter fluctuation amplitude parameter (S_8).

Early-universe measurements suggest $H_0 = 67.4 \pm 0.5$ km/s/Mpc and $S_8 = 0.831 \pm 0.013$, while late-time observations yield $H_0 = 73.04 \pm 1.04$ km/s/Mpc and $S_8 = 0.766^{+0.020}_{-0.014}$. These discrepancies correspond to tensions of approximately 5σ for H_0 and 3.1σ for S_8, respectively [9]. Various approaches have been proposed to overcome the shortcomings of the standard ΛCDM model, including dynamical dark energy, interactions between dark matter and dark energy, and radiation-based models. A compelling alternative is to modify gravity itself. Rather than introducing new matter fields or exotic energy components such as inflation or dark energy, modified gravity theories aim to explain both early- and late-time cosmic acceleration and structure formation through changes to the underlying laws of gravity. In recent decades, numerous such models have been developed to account for observations of galaxy and cluster dynamics, large-scale structure (LSS), Cosmic Microwave Background (CMB) anisotropies, and the present accelerated expansion, without invoking dark matter or dark energy [9, 10].

The idea of modifying gravity dates back to Einstein's later work, in which he attempted to unify GR and electromagnetism using affine connections, a concept introduced by mathematicians like Weyl. Although unification was not achieved, Einstein's approach gave rise to a formulation where gravity is mediated by torsion rather than curvature, which forms the foundation of metric teleparallel gravity. Later advancements showed that gravity could also emerge from non-metricity in flat, torsionless geometries. This led to the concept of the geometric trinity of gravity, which recognises three physically equivalent but geometrically distinct formulations of GR: curvature-based, torsion-based, and non-metricity-based, all rooted in metric-affine geometry. This trinity has attracted growing interest for its ability to provide new insights into the foundations of GR and address long-standing issues such as gravitational energy-momentum and black hole entropy [10]. In this study, we focus on a modified gravity model based on nonmetricity named $f(Q)$-gravity to explore its potential to resolve tensions H_0 and S_8.

2 f(Q) Cosmology

In $f(Q)$-gravity, the mathematical description between the geometry of spacetime and the distribution of matter can be obtained from the modified Einstein-Hilbert action [10]:

$$S_Q = \frac{1}{2\kappa} \int \sqrt{-g}\,[f(Q) + \mathcal{L}_m]\, d^4x, \tag{1}$$

where $\kappa = \frac{8\pi G}{c^4}$ with G being the Newtonian constant, g is the determinant of the metric $g_{\mu\nu}$, $\mathcal{L}_m$ is the matter lagrangian density, and $f(Q)$ is an arbitrary function of non-mentricity scalar Q defined as $Q = -Q_{\alpha\mu\nu}P^{\alpha\mu\nu}$. Furthermore, $Q_{\alpha\mu\nu} \equiv \nabla_\gamma g_{\mu\nu}$ and $P^{\alpha}_{\ \mu\nu} \equiv \frac{1}{4}\left(-Q^{\alpha}_{\ \mu\nu} + 2Q_{(\mu\ \ \nu)}^{\ \ \alpha} - Q^{\alpha}g_{\mu\nu} - \tilde{Q}^{\alpha}g_{\mu\nu} - \delta^{\alpha}_{(\mu}Q_{\nu)}\right)$ are the non-metric tensor with two independent traces ($Q_\alpha = Q_{\alpha\ \ \mu}^{\ \ \mu}$ and $\tilde{Q} = Q^{\mu}_{\alpha\mu}$) and the superpotential term, respectively [10, 11]. Furthermore, the Action (1) in a flat spacetime is equivalent to GR for $f(Q) = Q$. Now, setting $8\pi G = c^4 = 1$ to vary the action (1) with respect to the metric tensor and setting it to zero yields the following field equations:

$$\frac{2}{\sqrt{-g}}\nabla_\alpha(\sqrt{-g} f_Q P^{\alpha}_{\mu\nu}) = \frac{1}{2} f g_{\mu\nu} + f_Q(P_{\mu\alpha\beta}Q_{\nu}^{\alpha\beta} - 2Q_{\alpha\beta\mu}P^{\alpha\beta}_{\mu}) = T_{\mu\nu}, \tag{2}$$

where $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_m}{\delta g^{\mu\nu}}$ is the energy-momentum tensor,f is f(Q), and $f_Q \equiv \frac{\partial f}{\partial Q}$. For cosmological application, we consider the line element $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$ which corresponds to the spatially flat Friedman-Lemaitre-Robertson-Walker (FLRW) metric, in which δ_{ij} is the Kronecker delta and $a(t)$ is the cosmic scale factor used to define the Hubble expansion rate $H \equiv \frac{\dot{a}}{a}$ with a dot representing the derivative with respect to cosmic time. As a result, one assumes that the Universe is composed of a perfect fluid [12]:

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P q_{\mu\nu}, \tag{3}$$

such that the energy density and pressure, respectively, become

$$\rho = 6H^2 f_Q - \frac{1}{2}f, \tag{4}$$

$$P = \frac{1}{2}f - 2f_Q(\dot{H} + 3H^2 + 12H^2\dot{H}\frac{f_{QQ}}{f_Q}), \tag{5}$$

where $f_{QQ} \equiv \frac{\partial^2 f}{\partial Q^2}$ and the non-metric tensor becomes $Q = 6H^2$ then the modified Friedmann and Raychaudhuri equations are

$$3H^2 = \frac{1}{2f_Q}(\rho + \frac{f}{2}), \tag{6}$$

$$\dot{H} + 3H^2 = -\frac{1}{2f_Q}(24H^2\dot{H}f_{QQ} + P - \frac{f}{2}). \tag{7}$$

In this work, to solve the modified Friedmann and Raychaudhuri equation, we use the f (Q)-gravity model reconstructed using observation data without any prior ansatz on the underlying cosmological background by means of the Markov Chain Monte Carlo (MCMC) integration technique applied to the combined likelihood of the SNIa Pantheon sample and observational Hubble data from S. Capozziello and R. D'Agostino [13] as

$$f(Q) = \alpha + \beta Q^n, \tag{8}$$

where α, β, n are constant parameters that match the observational data after numerical reconstruction based on rational Pade approximations. The model matches GR when $\alpha = 0$ and $\beta = n = 1$, while it matches the ΛCDM model when $\alpha = -2\Lambda$ and $\beta = n = 1$. As a result, the partial derivative of the model with respect to Q is

$$f_Q = \beta n Q^{n-1}, \tag{9}$$

and using the fact that when the model in use was constructed, it was concluded that G_{eff} coincided with G at the present epoch translated to $f_Q = 1$ we get the following relation:

$$\beta = \frac{6H_0^2}{n(6H_0^2)^n}. \tag{10}$$

Assuming a non-relativistic matter and substituting Equations (7, 8, 9) into the modified Friedmann Equation (6), the normalised Hubble parameter ($h(z) = \frac{H(z)}{H_0}$) after some mathematical manipulations becomes

$$h(z) = \sqrt{\left(\frac{\Omega_{m,0}(1+z)^3 + \frac{\Omega_\alpha}{2}}{2 - \frac{1}{n}}\right)^{\frac{1}{n}}}, \tag{11}$$

where $\Omega_{m,0}$ is the dimensionless matter density parameter of the present value, and $\Omega_\alpha = \frac{\alpha}{3H_0^2}$ is a dimensionless parameter introduced in this study to ensure that $h(z)$ remains dimensionless. From Equation (11) we may deduce that $n \neq 0$ and $n > \frac{1}{2}$. As a result, in the next subsection we will use recent cosmological data to constrain Equation (11) and the flat ΛCDM model given as

$$h(z) = \sqrt{\Omega_{m.0}(1+z)^3 + (1 - \Omega_{m,0})}. \tag{12}$$

2.1 Cosmological observation

The data sets used are Baryon Acoustic Oscillations (BAO) measurements from the Dark Energy Spectroscopic Instrument (DESI) [14], cosmic chronometers (CC) from Hubble measurements [15], and SNIa distance moduli measurements from Pantheon+ &SH0ES (PantheonP+ SH0ES) [16] and using the Python package updated MCMC simulations named *Kosmulator* from R.T. Hough et al. [17] we can constrain the following paramaters $\Omega_{m,0}$, Ω_α, n, H_0, r_d, and M_{abs} following the work of S.Sahlu et al. [18] on how to put constraints on your model using these aforementioned datasets. Where r_d and M_{abs} are the sound horizon at the drag epoch with units Mpc and the calibrated absolute magnitude of an SNIa, respectively. Both r_d and H_0 have physical units; however, for the sake of brevity and neatness, we will present their values without explicitly including units from this point on. It should be understood that these quantities are not dimensionless. The free parameter to be constrained with their prior range are $[(0 \leq \Omega_m \leq 1), (0 \leq \Omega_\alpha \leq 2), (0.5 < n \leq 2), (0 \leq H_0 \leq 100), (100 \leq r_d \leq 200)$ and $(-22 \leq M_{abs} \leq -15)]$, while the free parameters with their known true value are $\Omega_m = 0.315$, $H_0 = 67.4$ $r_d = 147.05$ all three from Planck 2018 results [19] and $M_{abs} = -19.25$ from A.Mhamdi et al. [20]

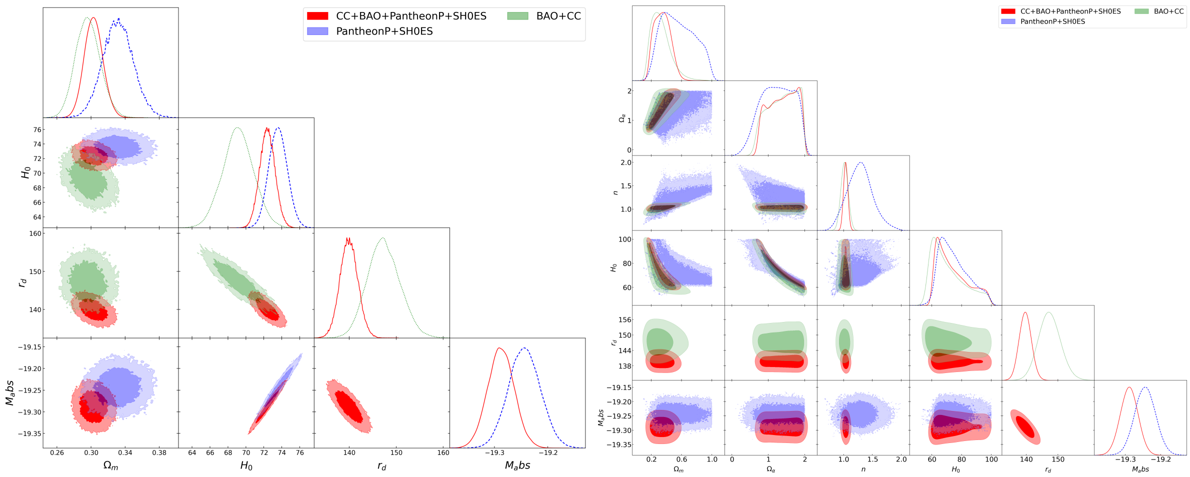

Figure 1: (*left*) Figure shows the contour plots and Gaussian graphs of the parameter distributions for the best-fit parameters of the ΛCDM mode, whereas (*right*) Figure presents the same for the $f(Q)$ model, using different combinations of CC, BAO, and SNIa (Pantheon+ SH0ES) datasets to refine the best-fitting values of the free parameters. Here, on this graphs Ω_m represent the value of $\Omega_{m,0}$, and the corresponding parameter values are all listed in Table 1.

Model	**Observation**	$\Omega_{m,0}$	Ω_α	n	H_0	r_d	M_{abs}
ΛCDM	CC+BAO+PantheonP+ SH0ES	$0.303^{+0.011}_{-0.011}$	—	—	$72.389^{+0.852}_{-0.844}$	$139.924^{+1.915}_{-1.869}$	$-19.288^{+0.025}_{-0.025}$
	PantheonP+ SH0ES	$0.332^{+0.018}_{-0.018}$	—	—	$73.579^{+1.036}_{-1.008}$	—	$-19.244^{+0.030}_{-0.029}$
	BAO+CC	$0.296^{+0.015}_{-0.014}$	—	—	$69.128^{+1.717}_{-1.700}$	$147.254^{+3.599}_{-3.320}$	—
$f(Q)$	CC+BAO+PantheonP+ SH0ES	$0.342^{+0.111}_{-0.113}$	$1.419^{+0.404}_{-0.471}$	$1.037^{+0.038}_{-0.037}$	$71.383^{+15.326}_{-8.071}$	$139.715^{+1.922}_{-1.908}$	$-19.289^{+0.025}_{-0.025}$
	PantheonP+ SH0ES	$0.543^{+0.267}_{-0.213}$	$1.263^{+0.489}_{-0.486}$	$1.284^{+0.215}_{-0.217}$	$73.283^{+12.813}_{-8.148}$	—	$-19.244^{+0.030}_{-0.029}$
	BAO+CC	$0.309^{+0.139}_{-0.108}$	$1.412^{+0.406}_{-0.483}$	$1.017^{+0.063}_{-0.063}$	$68.109^{+15.572}_{-8.064}$	$147.281^{+3.505}_{-3.345}$	—

Table 1: Best-fit parameter values for the ΛCDM and $f(Q)$ models using different combinations of datasets. The discussion and analysis of these results are explained in the Discussion and Conclusion section.

3 Discussion and Conclusion

According to the latest measurements from DESI BAO by A.G. Adame et al. [14] and the 2000 SNIa sample compilation from the Union Through UNITY project by D. Rubin et al. [21], the best-fit values for the matter density parameter are $\Omega_{m,0} = 0.295^{+0.015}_{-0.015}$ and $\Omega_{m,0} = 0.356^{+0.028}_{-0.028}$, respectively. A recent late-time constraint from the combined Pantheon+SH0ES datasets by D. Brout et al. [16] yields $\Omega_{m,0} = 0.334^{+0.018}_{-0.018}$, while the combination of BAO and cosmic chronometers (CC) of S. Sahlu et al. [18] gives $\Omega_{m,0} = 0.296^{+0.015}_{-0.014}$ within the flat ΛCDM model. A direct combination of Pantheon+SH0ES and early-time probes such as BAO must be approached with caution due to degeneracies, particularly in the absolute magnitude calibration of SNIa. As noted in the literature, such combinations are often avoided unless a model explicitly accounts for the overlapping information. These degeneracies can be mitigated by incorporating complementary measurements sensitive to both early- and late-time expansion histories, such as Planck's CMB data. This approach has been demonstrated in joint analyses by A.G. Adame et al. [14] and D. Brout et al. [16], which combined BAO, CMB and Pantheon + data to obtain model-consistent cosmological parameters.

In this work, we break the degeneracy between BAO and Pantheon+SH0ES by including CC data. This is evidenced by the nearly circular 2D contour plots, reduced uncertainties, and approximately Gaussian posterior distributions for the ΛCDM model, as shown in the left panel of Figure 1. From the DESI BAO and Union Through UNITY datasets, the values of Ω_m with their associated uncertainties under the flat Λ CDM model are summarised in Table 1. All combined data sets yield values within an acceptable range $0.28 \lesssim \Omega_m \lesssim 0.384$. This consistency also holds for the two combinations (CC+BAO+PantheonP+SH0ES and BAO+CC) under the $f(Q)$ model. However, the value of Ω_m obtained from the Pantheon+SH0ES dataset under the $f(Q)$ model lies outside this range, indicating a possible inconsistency.

Early-universe measurements suggest $H_0 = 67.4 \pm 0.5$, while late-time observations yield $H_0 = 73.04 \pm 1.04$ within the ΛCDM framework. This well-known tension is also evident in Table 1. Under the $f(Q) = \alpha + \beta Q^n$ model, the H_0 tension remains unresolved. In particular, while all data sets show large uncertainties in H_0 and slanted 2D contours with non-Gaussian distributions (see the right panel of Figure 1), the uncertainty in Ω_m is significantly large only in the Pantheon+SH0ES dataset (as seen in Table 1), potentially pointing to degeneracy in the current $f(Q)$ model. Consequently, when assuming $f_Q = 1$, the model exhibits degeneracies, and thus a full statistical or perturbative analysis is omitted in this work.

Interestingly, this same model has been studied without assuming $f_Q = 1$ by S. A. Narawade and B. Mishra [22] and by D. Mhamdi et al. [23]. Narawade and Mishra, using Hubble parameter and PantheonP+SH0ES data, reported best-fit values of $H_0 = 69.5^{+2.3}_{-1.9}$ and $H_0 = 70.7^{+2.7}_{-2.7}$, respectively. Mhamdi et al., combining PantheonP+SH0ES, CC, and redshift-space distortion measurements, obtained $H_0 = 71.65^{+0.84}_{-0.84}$. In both cases, the model was well constrained, with no signs of parameter degeneracy.

In conclusion, to rigorously evaluate the $f(Q)$ model as a candidate to resolve the H_0 and S_8 tensions and to perform a robust statistical assessment of its fit to cosmological data, a more comprehensive investigation is required. Since previous studies have not examined the compatibility of their constraints with both early- and late-time cosmological observations, this gap will be addressed in future work using a full range of datasets, without assuming $f(Q) = 1$. Furthermore, it is important to note that another valuable area of research is needed to verify the non-degeneracy that exists between BAO and Pantheon+ SH0ES data when combined with complementary measurements sensitive to early and late expansion histories.

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