A Power Law Model (PLM) approach to predict the performance of a small-scale PV system

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Abstract. This paper proposes a mathematical approach to predict the real-time performance of a small-scale rooftop photovoltaic (PV) system based on the power law model (PLM), commonly used to predict the I-V curves of solar cells under standard test conditions (STC). The shape parameters of the PLM, m and μ in this study, were determined using experimental data collected under normal weather conditions (irradiation and temperature) based on the Newton-Raphson algorithm iterative method. From the calculations performed on the MATLAB platform, obtained results reveal that the shape parameters have a weak dependence on both temperature and irradiation as shown by the low correlation coefficients of 0.296 and -0.110 respectively for μ and -0.201 and -0.188 respectively for m. It is however found that the shape parameters are strongly correlated to the electrical parameters: μ is strongly correlated to the fill factor with a correlation factor of 0.958 while m strongly depends on voltage with a correlation factor of 0.784. Additionally, this approach predicts the output electrical parameters of the PV system with high accuracy in real-time, with the mean values of R^2, RMSE, and correlation r of about 0.99, 3.07 %, and 0.99, respectively. Furthermore, the parameter μ is seen to vary between 0.8 and 1 during winter and from 0.5 to 0.8 in summertime, while m fluctuates between 10 and 20 during winter and from 10 to 25 during summertime. This approach offers high accuracy because the shape parameters take into account the dynamical behavior of the losses (internal and external), such as recombination effects, series and shunt resistance in each PV module, which vary under different weather conditions. The limitation of the model is that there is not enough information to link the shape parameters to other electrical parameters such as ideal factor, saturation current, shunt and series resistance to perform the qualitative investigations based on these parameters.

1. Introduction

Renewable energy has the potential to be a sustainable solution capable of tackling both the energy crisis and climate issues [1-3]. Among renewable energy sources, photovoltaic (PV) energy appears to be a promising candidate that can satisfy the energy demand due to its long lifetime of about 20 years, environmental friendliness, and cost effectiveness. However, the main challenge with PV energy is that it depends totally on weather conditions (irradiation, temperature, humidity, wind, etc.) which vary randomly and make the prediction of power production difficult [4-6]. Energy produced by PV modules has been estimated in the past decade through the mathematical modelling of PV modules with the single-diode model (SDM) [7-9], and double-diode model (DDM) [10]. Their approaches are based on extracting the electrical parameters (short circuit current, saturation current, ideality factor, shunt, and series resistance) under standard test condition (STC). However, this approach offers high accuracy only for STC as defined by the manufacturer because the parameters extracted are fixed, but it offers poor prediction outdoors in real-time monitoring. Hao and co-workers [11] recently proposed a mathematical model of the losses (shunt and series resistance), saturation current, and ideality factor, which vary with the weather conditions to predict characteristic I-V curves, albeit with low accuracy. Karmalkar et al., proposed the Power Low Model (PLM) based on shape parameters to improve the prediction of I-V curves of solar cells under STC [12]. Zhang et al., combined PLM and SDM to predict the I-V curve of a solar cell also under STC [13]. From the works mentioned above, we remark that the extraction of the shape parameters was carried out (only for STC) by proposing different mathematical models to predict the I-V curve of a solar cell. Thus, this work aims to extract the shape parameters for real-time conditions and not just for a solar cell but rather for a small-scale PV system by using real-time weather conditions collected to predict in real-time the performance (current, voltage, maximum power) of a typical small-scale PV system (~ 2 kW). The content of the work is organized as follows: Section 2 lays out the mathematical background , describing the various parameters involved in PLM. The various mathematical models, such as short circuit current and open circuit voltage, are established, leading to the extraction of shape parameters based on the Newton-Raphson method. Section 3 is dedicated to the presentation of the variation in shape and electrical parameters over time, closed by a conclusion and limitations.

2. Mathematical modelling procedure

The power law model (PLM) is made of four parameters including the short-circuit current, open circuit voltage, and the two shape parameters μ and m . The normalized I-V characteristics curves of PLM its expressed as [12]

$$i = 1 - \left[1 - \mu(G,T)\right] v - \mu(G,T) v^{m(G,T)} \quad (1)$$

wherein $i = I/I_{sc}$ and $v = V/V_{oc}$ are normalized current and voltage, I and V are current and voltage output of the device while $\mu(G,T)$ and $m(G,T)$ are the shape parameters. From Eq.(1), the short-circuit I_{sc} and open circuit voltage V_{oc} are expressed as follows [14]

$$I_{sc} = \frac{G}{G_{ref}} \left[I_{scref} + K_i \left(T - T_{ref} \right) \right] \quad (2)$$

$$V_{oc} = V_{ocref} - K_V \left(T - T_{ref} \right) \quad (3)$$

Wherein I_{scref} and V_{ocref} are the short circuit current and open circuit voltage of the PV system under standard test condition (STC) at the reference temperature T_{ref} and G_{ref} is the reference irradiation normalized by 1000 W/m^2. G and T represent the irradiation and the solar panel temperature obtained from the experiment within the day, K_i and K_V represent the temperature constant of photocurrent and voltage, respectively. Knowing that the normalized power is expressed as $p = i \cdot v$, at the maximum power we have:

$$\frac{\partial p}{\partial v} = v_p \left. \frac{\partial i}{\partial v} \right|_{i=i_p, v=v_p} + i_p = 0 \quad (4)$$

After rearrangement, Eq. (4) can be rewritten as

$$\left. \frac{\partial i}{\partial v} \right|_{i=i_p, v=v_p} = -\frac{i_p}{v_p} \quad (5)$$

Differentiating Eq. (1) and substituting it into Eq. (4), we have

$$\mu = \left(-\frac{i_p}{v_p}+1\right)\left(\frac{1}{1-m\cdot v_p^{m-1}}\right) \tag{6}$$

wherein i_p and v_p define the dimensionless current and voltage at the maximum power. Equations (1) and (6) form a system of nonlinear equations that can be solved using the Newton-Raphson iteration method in order to extract the shape parameters for various weather conditions in real-time expressed as follows

$$m_{k+1} = m_{k+1} - \frac{f\left(m_{k+1}\right)}{f_p\left(m_{k+1}\right)} \tag{7}$$

wherein the objective function $f\left(m^k\right)$ its expressed as follows

$$f\left(m_{k+1}\right) = -i_p + 1 - \left[1-\mu\left(G,T,m_{k+1}\right)\right]v_p - \mu\left(G,T,m_{k+1}\right)v_p^{m_{k+1}} \tag{8}$$

The derivative function of f denoted f_p its expressed as

$$f_p = \frac{\partial\mu}{\partial m_{k+1}}v_p - \frac{\partial\mu}{\partial m_{k+1}}v_p^{m_{k+1}} - \mu\cdot\log\left(v_p\right)\cdot v_p^{m_{k+1}} \tag{9}$$

Then the final formula implemented in the Newton-Raphson algorithm is expressed as

$$m_{k+1} = m_k - \frac{-i_p + 1 - \left[1-\mu\left(G,T,m_k\right)\right]v_p - \mu\left(G,T,m_k\right)v_p^{m_k}}{\frac{\partial\mu}{\partial m_k}v_p - \frac{\partial\mu}{\partial m_k}v_p^{m_k} - \mu\cdot\log\left(v_p\right)\cdot v_p^{m_k}} \tag{10}$$

wherein $\frac{\partial\mu}{\partial m_k} = \left[-\frac{i_p}{v_p}+1\right]\left(\frac{v_p^{m_k-1} + m_k\cdot\log\left(v_p\right)\cdot v_p^{m_k-1}}{\left(1-m_k\cdot v_p^{m_k-1}\right)^2}\right)$ and k represents the iteration order.

The resolution of Eq. (10) allows for determination of the shape parameters and so prediction of the PV system's real-time performance.

3. Methodology

The small 2 kW PV system installed on the rooftop of the Arcadia campus of the Tshwane University of Technology in Pretoria, South Africa was used as a test bed in this study. The irradiation and the temperature were measured as weather parameters to monitor the PV system dynamics in real time using the PLM approaches. The PV system is made of two strings of PV modules of monocrystalline silicon half-cut cells, each containing 12 identical PV modules connected in series. The data collection was conducted over 17 days, from 11 a.m. to 2 p.m., to avoid shading created by an adjacent building on the panels from 8 a.m. to 11 a.m. and 2 p.m. to 6 p.m. The step collection of the data was set to 15 minutes. This work presents variations in shape parameters and compares experimental and predicted data from PLM.

Figure 1: Overview image of the small-scale PV system

The experiment was done during the winter and summer periods to check the behavior of this system under different weather conditions. The output electrical parameters, such as power, fill factor, current, and voltage, obtained through the experiment at the maximum power, were compared with the same parameters obtained using PLM

4. Results and discussion

4.1. Variation of the shape parameters.

This subsection focuses on the variation of shape parameters extracted using the experimental data as shown in Fig 1. The graphs present two zones, first pink for the winter period while green for summertime.

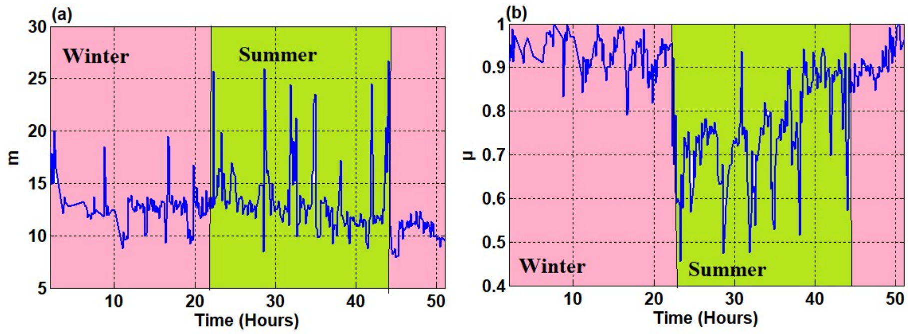

Figure 2: variation of the shape parameters versus time

The graphs show that the two parameters tend to be vary in opposite directions. In fact, during wintertime, m and μ vary between 10 to 15 and 0.8 to 1, respectively while during summertime they vary between 10 to 25 and 0.5 to 0.8. This shows that during winter, the slope around the short circuit situation is conserved and the shape of the I-V curve is preserved, wherein μ is around 1 and m is closer to 10 [13]. In contrast, during summertime, the slope is steeply downward at the short circuit position because of the low value of μ, leading to the loss of the I-V curve shape. In this situation, the quality of the device is altered due to the recombination that occurs due to the high temperature of the PV module. Thus, m and μ can be used both to check the quality and to predict the performance of the PV cell, module and system in real-time, as shown in the correlation on **Table 1**.

Table 1: Correlation table between the data.

	I_p	V_p	FF	P_p	G	T	μ	m
I_p	**1**							
V_p	**-0.567**	**1**						
FF	**-0.023**	**-0.024**	**1**					
P_p	**-0.439**	**-0.439**	**0.490**	**1**				
G	**0.814**	**-0.506**	**-0.130**	**0.794**	**1**			
T	**0.746**	**-0.631**	**0.296**	**0.695**	**0.628**	**1**		
μ	**0.478**	**-0.236**	**0.958**	**0.485**	**-0.110**	**0.296**	**1**	
m	**-0.427**	**0.784**	**-0.282**	**-0.334**	**-0.188**	**-0.201**	**-0.511**	**1**

4.2. *Variation of the output electrical parameters*

This part focuses on the variation of the output electrical parameters, known as current, voltage, fill factor, and power at the maximum operating point obtained through PLM and experiment. The number of peaks shown in Figs. 3(a) and (d) define the number of days of the experiment. They reflect the maximum incident irradiation received by the PV system. The graphs are divided into two periods: winter and summer. We noticed a good fit of the PLM approaches with the experiment due to the performance metrics displayed in **Table 2**.

Table 2: Performance metric

	Fill factor	Current (A)	Voltage (V)	Power (W)	Mean value
R^2	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**
RMSE	**0.005%**	**0.2%**	**0.07%**	**12%**	**3.07%**
r	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**

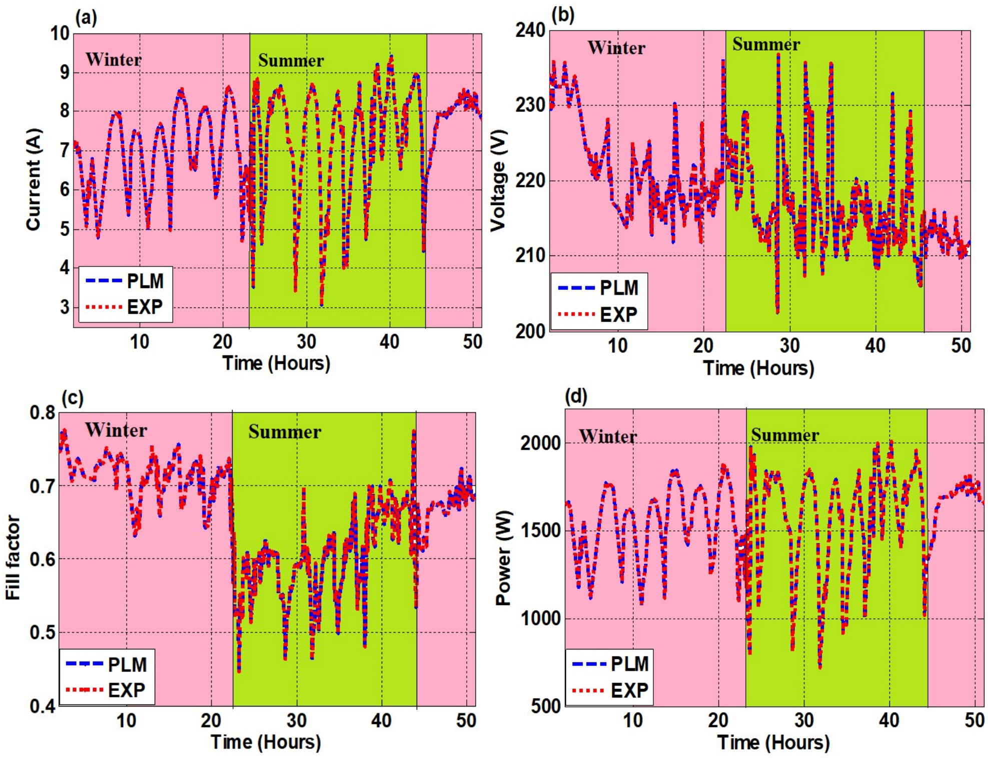

Figure 3: Variation of the electrical parameters versus time (a) current, (b) voltage (c) fill factor (d) power

During winter (pink band), the fill factor variations inf orm us that the I-V curve shape is preserved, due to the high value oscillating around 0.7, but the output current and power are relatively low, while the voltage is high. In fact, during wintertime, small-scale PV systems are subjected to a relatively low irradiation level, leading to a low environmental temperature. Thus, the thermal agitation of the carriers in each PV module is minimized because of natural cooling. Then, the low incident irradiation during winter will excite many carriers, but few are extracted. Thus, the carriers stand at the PN junction to increase the voltage as depicted in Fig 3 (b). During summertime, we have high power and current due to high irradiance levels, but the fill factor and the voltage are relatively low. This behaviour shows that the losses become non-negligible at high irradiance (high temperature) due to the decrease of the fill factor and voltage. Thus, these losses altered the quality of the PV system by reducing the fill factor, leading to the presence of the recombination effect in the system. Additionally, we notice from **Table 1** that μ and m are strongly correlated to the fill factor and voltage. This means that the quality of the system can be checked through μ while the variation of the voltage can be investigated using m.

5. Conclusion

This paper aimed to propose a mathematical model to predict the performance of a small-scale PV system using the power law model (PLM) commonly used to predict the I-V characteristics of solar cells in standard test conditions (STC). The shape parameters (m, μ), involved in the modeling of PLM are extracted using experimental data (irradiation, module temperature, power, current, and voltage) collected in outdoor conditions for 17 days. The results reveal that the shape parameters depend more on output electrical parameters than on weather conditions (irradiation and temperature). We also found that this model offers a good fit because of R^2, correlation (r), and RMSE of about 0.99, 3.07 %, and 0.99, respectively. Additionally, this model (PLM) offers high performance because it considers the dynamic behavior of the losses (resistance, recombination effect) in the system, which varies under weather conditions. The limitations of this model are such that the shape parameters can only be known once the experimental data are collected. Additionally, this model does not give much information about the behaviour of the electrical parameters in real-time, such as ideality factor, saturation current, shunt and series resistance, to monitor internally the behaviour of the PV system.

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References

[1] Kapim Kenfack AD, Msimanga M, 2025 *Next Material*, **6** 1.
[2] Kapim Kenfack AD, Msimanga M, 2025, *J. Electron Mater*. **54** 1278.
[3] Kapim AD, Nangmetio CA, and Kuatche NJ, 2023. *J. Electron. Mater*. **52** 951.
[4] Zhang HY, Lu H, Gu T, Yang M., 2022. *Int. J. Electr. Power Energy Syst*. **134** 1.
[5] Pal RS, Mukherjee V, 2021. *Energy Convers. Manage*. **244** 1.
[6] Pelap FB, Tagne EK, and A.D.K. Kenfack 2021. *J. Ren. En.* **24**, 25
[7] Cuce E, Cuce PM., Karakas IH, Bali T, 2017. *Energy Convers. Manage*. **146**, 205.
[8] Hao P, Zhang Y, Lu H, Lang Z, 2021. *Energy Convers. Manage*. **247** 1.
[9] Hao P, Zhang Y, 2021. *IEEE J. Photovolt.* **11 (6),** 1446.
[10] Chin VJ, Salam Z, Ishaque K, 2016. *Energy Convers. Manage*. **124**, 42.
[11] Hao L, Yunpeng Z, Peng H, Jiao M, Li Z, Tingkun G, Ming Y, 2022 *Energy Reports,* **8** 13859
[12] Karmalkar S, Saleem H, 2011. *Solar Energy Materials & Solar Cells* **95** 1076.
[13] Zhang C, Zhang Y, Su J, Gu T, Yang M, 2020. *IEEE J. Photovolt.* **10 (6)**, 1816.
[14] Thabiso N, Thantsha NM, Kapim Kenfack AD, Msimanga M, 2024, *Proceeding of SAIP2024, 68th Annual conference of the South Africa Institute of Physics*, pp 569-575, ISBN:978-1-0370-2645-4.